

Inflation in a scalar-vector-tensor theory

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Abstract

In this work, we study inflation in a particular scalar-vector-tensor theory of gravitation without the $U(1)$ gauge symmetry. The model is constructed from the more general action introduced in Heisenberg et al. (Phys Rev D 98:024038, 2018) using certain specific choices for the Lagrangians and the coupling functions. Also, for this model we build the explicit form for the action, and from it, we derive the general equations: the energy-momentum tensor and the equations of motion, and using the flat FLRW background, we have analyzed if it's possible to obtain an inflationary regime with it. Additionally, using particular choices for the potential, the coupling functions, suitable dimensionless coupling constants and initial conditions, it was possible to verify numerically that this model of inflation is viable. In this sense, we could verify that the introduction of the coupling function $f(\phi)$ in our model of inflation, allows us to reach a suitable amount of e -foldings N for sufficient inflation. This is a remarkable result, since without the coupling function contribution, the amount of e -foldings is smaller at the end of inflation, as has been demonstrated in Heisenberg et al. (2018). Also, the no-ghosts and stability conditions that the model during inflation must satisfy, i.e., absence of ghosts and Laplacian instabilities of linear cosmological perturbations were obtained, furthermore these conditions were verified numerically too.

Keywords: Inflation; scalar-vector-tensor theories; Cosmology, No-ghosts and stability conditions; Linear cosmological perturbations

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1 Introduction

The hypothesis of inflation is currently considered as a part of the standard modern cosmology. This hypothesis was introduced in the early 1980's to resolve some problems that the Hot Big Bang model of the universe can't explain (e.g., the horizon, the flatness, and the monopole problems, among others) [1, 2, 3, 4]. Moreover, the inflationary paradigm allows us to make predictions about properties of the current universe

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which are in excellent agreement with the cosmological and astrophysical observations (e.g., the temperature fluctuations in CMB spectrum [5, 6, 27], the existence of large scale structures [8, 9, 10, 11], and the nearly scale invariant primordial power spectrum [12, 13, 14]).

The origin or source of inflation is still now unknown. However, there are many proposals in the literature to explain the inflationary regime. For example, models with a scalar field (the so called inflaton field) minimally coupled to gravity and with a nearly flat potential have been considered [15, 16]. In general, the inflaton field it can be considered non-minimally coupled to gravity too (via the Ricci scalar, the Ricci tensor, the Gauss-Bonnet invariant, etc). In this sense, the resulting models are known as scalar-tensor theories of gravitation. Others proposals include Einstein–aether, and Bimetric theories, as well as TeVeS, $f(R)$, general higher-order theories, Hořava–Lifschitz gravity, Galileons, Ghost Condensates, and models of extra dimensions including Kaluza-Klein, Randall–Sundrum and DGP (see e.g. Refs. [17, 18, 19] for a review). In addition, in recent years a considerable amount of works have been considered to model the inflationary scenario using vector fields, e.g. the generalized Proca fields and extensions thereof, or several Proca fields, as well as bigravity theories and scalar-vector-tensor (SVT) theories (for more details about these topics see Ref. [20] and references therein). The SVT theories were introduced in Ref. [21] and them constitute a consistent ghost-free covariant gravity theories with second order equations of motion with derivative interactions. These SVT theories have important implications for cosmological and astrophysical applications [22, 23, 24, 25, 26].

In this work we focused in study inflation in a particular SVT theory of gravitation without the $U(1)$ gauge symmetry. This model is constructed from the more general action presented in Ref. [23] using certain specific choices for the Lagrangians and the coupling functions. Also, we derive the no-ghosts and stability conditions that the model during inflation must satisfy, i.e. absence of ghosts and Laplacian instabilities of linear cosmological perturbations.

This paper it is organized as follows: in section 2 we introduce the scalar-vector-tensor model of inflation, and for this model we construct the explicit form for the action, from which, we derive the general equations: the energy-momentum tensor and the equations of motion. In section 3, we write the general equations obtained in section 2 for a concrete background (the flat FLRW background), and in this way, we verify if it is possible to obtain an inflationary regime with this model. Also, in this section, using concrete choices for the potential, the coupling functions, suitable dimensionless coupling constants and initial conditions, we verify numerically if this model of inflation is viable. In section 4, the no-ghosts and stability conditions that the model during inflation must satisfy, i.e., absence of ghosts and Laplacian instabilities of linear cosmological perturbations, are obtained, moreover these conditions are verified numerically too. Finally, some conclusions are exposed in section 5.

2 The model

In this section, we shall present in brief some basic features of the SVT theories, and also, we derive the general equations for the model under consideration.

The most general SVT model with broken $U(1)$ gauge symmetry which is ghost-free, with second order equations of motion and with derivative interactions is given by the action [23]

$$S_{SVT} = \int d^4x \sqrt{-g} \sum_{n=2}^6 \mathcal{L}_n, \quad (2.1)$$

where the Lagrangians $\mathcal{L}_2, \dots, \mathcal{L}_6$, are given by

$$\begin{aligned} \mathcal{L}_2 &= f_2(\phi, X_1, X_2, X_3, F, Y_1, Y_2, Y_3), \\ \mathcal{L}_3 &= f_3(\phi, X_3)g^{\mu\nu}S_{\mu\nu} + \tilde{f}_3(\phi, X_3)A^\mu A^\nu S_{\mu\nu}, \\ \mathcal{L}_4 &= f_4(\phi, X_3)R + f_{4,X_3}(\phi, X_3)[(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu], \\ \mathcal{L}_5 &= f_5(\phi, X_3)G^{\mu\nu}\nabla_\mu A_\nu - \frac{1}{6}f_{5,X_3}(\phi, X_3)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] \\ &\quad + \mathcal{M}_5^{\mu\nu}\nabla_\mu \nabla_\nu \phi + \mathcal{N}_5^{\mu\nu}S_{\mu\nu}, \\ \mathcal{L}_6 &= f_6(\phi, X_1)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \mathcal{M}_6^{\mu\nu\alpha\beta}\nabla_\mu \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi + \tilde{f}_6(\phi, X_3)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \mathcal{N}_6^{\mu\nu\alpha\beta}S_{\mu\alpha}S_{\nu\beta}, \end{aligned} \quad (2.2)$$

This model was originally introduced in [21] and its cosmological implications were studied in [23]. Moreover, in [22] the authors using this model with gauge-invariant derivative scalar-vector interactions and a non-minimal coupling to gravity, to analyze the properties of black holes on a static and spherically symmetric background. Also, in [24] the cosmology of SVT theories with parity invariance was studied, paying particular attention to the application to dark energy. Finally, in [25] the gauge-ready formulation of cosmological perturbations on the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background was performed (taking into account a matter perfect fluid).

In concrete, in this work we consider the following special case for the action given by Eq. (2.1)

$$S = \int d^4x \sqrt{-g}(\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_5), \quad (2.3)$$

where \mathcal{L}_2 , \mathcal{L}_4 and \mathcal{L}_5 are construct considering the following particular choices for f_2 , f_4 , and f_5 in Eq. (2.2):

$$\begin{aligned} f_2 &= F + X_1 - V(\phi) + \beta_m M X_2 + \beta_A M^2 X_3, \\ f_4 &= \frac{M_{pl}^2}{2} + \beta_G X_3, \\ f_5 &= f(\phi), \end{aligned} \quad (2.4)$$

and $X_1 = -\frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi$, $X_2 = -\frac{1}{2}A^\mu \nabla_\mu \phi$, $X_3 = -\frac{1}{2}A_\mu A^\mu$, $F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, ($F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$), $V(\phi)$ is a scalar potential, M is a constant having a dimension of mass, M_{Pl} is the reduced Planck mass, and β_m , β_A , β_G are dimensionless coupling constants. $f(\phi)$ is an arbitrary coupling function. The other quantities and coupling functions that appear in Eq. (2.2) are defined in [23], but in our case they are taken to be 0. A

model of inflation using f_2 and f_4 was analyzed in [23]. The introduction of the coupling function $f_5 = f(\phi)$ in our model induces a new mixing between the scalar field and the vector field via the Einstein tensor (in addition to the term $\beta_m M X_2$ of f_2). On the other hand, it could have significant implications onto the dynamics of inflation. For example, the amount of e -foldings N obtained with the model introduced in [23] is not sufficient in the end of inflation. In this way, with our model we will try to improve the amount of e -foldings N , among others calculations. This analysis and others we will carry out in the next sections.

Now, from the above, the action given by Eq. (2.3) takes the following explicit form:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \beta_A M^2 A_\mu A^\mu - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) - \frac{1}{2} \beta_G A_\mu A^\mu R + \beta_G [(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu] - \frac{1}{2} \beta_m M A^\mu \nabla_\mu \phi + f(\phi) G_{\mu\nu} \nabla^\mu A^\nu \right], \quad (2.5)$$

However, taking into account the identity

$$\int d^4x \sqrt{-g} [(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu] = \int d^4x \sqrt{-g} R_{\mu\nu} A^\mu A^\nu, \quad (2.6)$$

the action Eq. (2.5) is reduced to

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \beta_A M^2 A_\mu A^\mu - \frac{1}{2} \beta_m M A^\mu \nabla_\mu \phi + \left(\beta_G A^\mu A^\nu + f(\phi) \nabla^\mu A^\nu \right) G_{\mu\nu} \right], \quad (2.7)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor. We can see in Eq. (2.7) that the action S can be written as $S = S_{HE} + S_\phi + S_A + S_{\phi A}$, where S_{HE} stands the Hilbert–Einstein action, S_ϕ represents the pure scalar field sector of the action, S_A includes the pure vector field sector and a term coupled to the Einstein tensor. $S_{\phi A}$ denotes the sector which include the coupling between the scalar field ϕ and the vector field A^μ (via derivative scalar-vector interactions) along with a term coupled to the Einstein tensor too. From this last sector, we will hope obtain new contributions to the background equations that allow us study the inflationary paradigm.

Since we have a explicit form for the action, Eq. (2.7), then in this paper we will employ the usual algorithm to obtain the general equations of motion, which is different to that used in [23], however, the results are equivalent.

The variation of the action Eq. (2.7) with respect to the metric tensor $g_{\mu\nu}$ gives the field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (2.8)$$

where $T_{\mu\nu}$ is the energy–momentum tensor which is constructed from $S_\phi + S_A + S_{\phi A}$, and has the following form:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} := T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(A)} + T_{\mu\nu}^{(A\phi)}, \quad (2.9)$$

Each term is given by the following expressions:

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \nabla_\alpha \phi \nabla^\alpha \phi + V(\phi) \right), \quad (2.10)$$

$$T_{\mu\nu}^{(A)} = F_{\mu\alpha} F_\nu^\alpha + \beta_A M^2 A_\mu A_\nu + \beta_G \left(R_{\mu\nu} A_\alpha A^\alpha - 4G_{(\mu\alpha} A_{\nu)} A^\alpha - R A_\mu A_\nu + 2\nabla_\alpha \nabla_{(\mu} (A_{\nu)} A^\alpha) - \square (A_\mu A_\nu) - \nabla_\mu \nabla_\nu (A_\alpha A^\alpha) \right) - g_{\mu\nu} \left[\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} \beta_A M^2 A_\alpha A^\alpha + \beta_G \left(\nabla_\alpha \nabla_\beta (A^\beta A^\alpha) - \square (A_\alpha A^\alpha) - G_{\alpha\beta} A^\alpha A^\beta \right) \right], \quad (2.11)$$

$$T_{\mu\nu}^{(A\phi)} = \beta_m M A_{(\mu} \nabla_{\nu)} \phi + f(\phi) \left[R_{\mu\nu} \nabla_\alpha A^\alpha - R \nabla_{(\mu} A_{\nu)} - 2G_{(\mu\alpha} \nabla_{\nu)} A^\alpha \right] + 2G_{(\mu\alpha} A_{\nu)} \nabla^\alpha f(\phi) - \nabla_\alpha [f(\phi) G_{\mu\nu} A^\alpha] - \nabla_\mu \nabla_\nu [f(\phi) \nabla_\alpha A^\alpha] + \nabla_\alpha \nabla_{(\mu} [f(\phi) \nabla_{\nu)} A^\alpha] + \nabla_\alpha \nabla_{(\mu} [f(\phi) \nabla_{\nu)} A^\alpha] - \square [f(\phi) \nabla_{(\mu} A_{\nu)}] - g_{\mu\nu} \left[\frac{1}{2} \beta_m M A^\alpha \nabla_\alpha \phi - f(\phi) G_{\alpha\beta} \nabla^\alpha A^\beta + \nabla_\alpha \nabla_\beta [f(\phi) \nabla^\beta A^\alpha] - \square [f(\phi) \nabla_\alpha A^\alpha] \right]. \quad (2.12)$$

On the other hand, the variation of the action with respect to ϕ gives the equation of motion,

$$\square \phi - V_{,\phi} + \frac{1}{2} \beta_m M \nabla_\mu A^\mu + G_{\mu\nu} \nabla^\mu A^\nu f_{,\phi} = 0, \quad (2.13)$$

where $f_{,\phi}$ denotes derivative of f with respect to ϕ . In a similar way, the equation of motion for the vector field is given by

$$\nabla^\nu F_{\mu\nu} + \frac{1}{2} \beta_m M \nabla_\mu \phi + \beta_A M^2 A_\mu - \left(2\beta_G A^\nu - \nabla^\nu f(\phi) \right) G_{\mu\nu} = 0. \quad (2.14)$$

3 Inflation in a SVT theory

In this section we use the general equations obtained above in a concrete background and in this way we will see if possible to obtain a inflationary regime with this model. The usual choice for the background is given by the flat Friedmann-Robertson-Walker (FRW) metric whose line element has the following form

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (3.1)$$

where $a(t)$ is the scale factor. In order to guarantee the homogeneity and isotropy of the universe, in the next calculations, we will regard that the vector field is without spatial components, i.e.

$$A_\mu(t) = (A_0(t), 0, 0, 0), \quad (3.2)$$

and the scalar field is time dependent only, $\phi = \phi(t)$. Thereby, using the FRW metric and the vector and scalar fields defined previously, the Einstein's equations Eq. (2.8) and the equations of motion Eqs. (2.13) and (2.14) are reduced to

$$3M_{\text{Pl}}^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2} \beta_A M^2 A_0^2 + 9\beta_G H^2 A_0^2 + \frac{1}{2} \beta_m M A_0 \dot{\phi} - 9H^2 A_0 \dot{\phi} f_{,\phi}, \quad (3.3)$$

$$-(2\dot{H} + 3H^2) M_{\text{Pl}}^2 = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \beta_A M^2 A_0^2 - 3\beta_G H^2 A_0^2 - 4\beta_G H A_0 \dot{A}_0 - 2\beta_G \dot{H} A_0^2 + \frac{1}{2} \beta_m M A_0 \dot{\phi} + 3H^2 A_0 \dot{\phi} f_{,\phi} + 2H \dot{A}_0 \dot{\phi} f_{,\phi} + 2\dot{H} A_0 \dot{\phi} f_{,\phi} + 2H A_0 \ddot{\phi} f_{,\phi} + 2H A_0 \dot{\phi}^2 f_{,\phi\phi}, \quad (3.4)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \frac{1}{2}\beta_m M \left(\dot{A}_0 + 3HA_0 \right) - 3H \left(3H^2 A_0 + H\dot{A}_0 + 2\dot{H}A_0 \right) f_{,\phi} = 0, \quad (3.5)$$

$$\frac{1}{2}\beta_m M \dot{\phi} + \beta_A M^2 A_0 + 3H^2 \left(2\beta_G A_0 - \dot{\phi} f_{,\phi} \right) = 0. \quad (3.6)$$

It is clear that the introduction of the coupling function $f(\phi)$ in our model gives rise to new and important terms in the background equations (although they are more complicated). Additionally, making suitable choices for $V(\phi)$, $f(\phi)$, the dimensionless coupling constants β_m , β_A , β_G and appropriated initial conditions, is possible in principle, to obtain a viable inflationary regime with this model.

Now, from Eq. (3.6), we can write the temporal vector component A_0 as

$$A_0 = \frac{-\beta_m M + 6H^2 f_{,\phi}}{2(\beta_A M^2 + 6\beta_G H^2)} \dot{\phi}, \quad (3.7)$$

Is evident from Eq. (3.7) that there is a direct explicit relation between the temporal vector component A_0 and $\dot{\phi}$, and an implicit relation with ϕ (through of the coupling function $f(\phi)$). Therefore, in this case we can say that the dynamics of inflation is driven by the scalar field ϕ and its derivative $\dot{\phi}$. If we consider that

$$f_{,\phi} = \frac{\partial f}{\partial \phi} = \frac{\beta_m M}{6H^2}, \quad (3.8)$$

then $A_0 = 0$ and the model corresponds to that with a single scalar field (inflaton) minimally coupled to gravity, i.e. the action Eq. (2.7) it reduces to $S_{HE} + S_\phi$, which has been widely studied in the literature. Now, considering a pure de Sitter expansion for the inflationary stage, we can found the coupling function $f(\phi)$ integrating Eq. (3.8) (since H is constant in this case)

$$f(\phi) = \frac{\beta_m M}{6H^2} \phi + C, \quad (3.9)$$

where C is an integration constant.

In order to analyze the inflationary regime with the background Eqs. (3.3)-(3.6), we consider the slow-roll parameters defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad (3.10)$$

additionally, during the inflationary phase these slow-roll parameters must satisfy the restriction $|\epsilon| \ll 1$, $|\epsilon_V| \ll 1$ and $|\eta| \ll 1$ [27]. Besides, in our case we define the following slow-roll parameters

$$|\delta_1| \equiv \left| \frac{A_0 \ddot{\phi} f_{,\phi}}{H} \right| \ll 1, \quad |\delta_2| \equiv \left| \frac{A_0 \dot{\phi}^2 f_{,\phi\phi}}{H} \right| \ll 1. \quad (3.11)$$

Replacing Eq. (3.7) and its time derivative into Eq. (3.4) and using Eq. (3.3) to eliminate V , the slow-roll parameter ϵ takes the form

$$\begin{aligned} \epsilon = & \left\{ \dot{\phi}^2 (\beta_A M^2 + 6\beta_G H^2) \left[\beta_A M^4 (4\beta_A - \beta_m^2) + 2H \left(M^2 (24\beta_A \beta_G - \beta_G \beta_m^2 (3 + 2\eta)) + 2M \beta_A \beta_m (3 - 2\eta) f_{,\phi} H \right. \right. \right. \\ & + 6(12\beta_G^2 + M f_{,\phi} (6\beta_G \beta_m + \beta_A M (4\eta - 3) f_{,\phi})) H^3 + 36\beta_G (2\eta - 3) f_{,\phi}^2 H^5 + 2(-\beta_A \beta_m M^3 + 12f_{,\phi} (M^2 \beta_A H^2 \\ & + 3\beta_G H^4)) f_{,\phi\phi} \dot{\phi} \left. \left. \left. \right] \right\} / \left\{ H^2 \left[8M_{\text{Pl}}^2 (\beta_A M^2 + 6\beta_G H^2)^3 + 2 \left(-M^4 \beta_A \beta_m (\beta_G \beta_m + 2M \beta_A f_{,\phi}) + 18M^2 (\beta_G^2 \beta_m^2 \right. \right. \right. \\ & + 2M \beta_A f_{,\phi} (\beta_G \beta_m + M \beta_A f_{,\phi})) H^2 + 108M^2 \beta_A \beta_G f_{,\phi}^2 H^4 + 216\beta_G^2 f_{,\phi}^2 H^6 \left. \left. \left. \right] \dot{\phi}^2 \right\}, \end{aligned} \quad (3.12)$$

where we have used the slow-roll parameter η to eliminate $\ddot{\phi}$, and $f_{,\phi\phi}$ depicts the second derivative of f with respect to ϕ .

Now, considering in Eqs. (3.5) and (3.7) the slow-roll parameters given by Eqs. (3.10) and (3.11) in an inflationary regime, then we can obtain an approximate expression for $\dot{\phi}$

$$\dot{\phi} \approx \frac{4V_{,\phi}(M^2\beta_A + 6\beta_G H^2)}{3H \left[M^2(\beta_m^2 - 4\beta_A) - 12(2\beta_G + M\beta_m f_{,\phi})H^2 + 36f_{,\phi}^2 H^4 \right]}, \quad (3.13)$$

For $f(\phi) = 0$ the Eqs. (3.12) and (3.13) it reduces to those obtained in [23]. For suitable values of the dimensionless coupling constants β_m , β_A , β_G and some specific choices for the coupling function $f(\phi)$, we can assume that the scalar potential V dominates over the other terms on the right-hand side of Eq. (3.3) during slow-roll inflation. This assumption has sense, since all terms on the right-hand side of Eq. (3.3) (apart of V) are proportional to $\dot{\phi}^2$ (after of replacing A_0 by Eq. (3.7)) and as is well known, in the inflationary regime $\dot{\phi}^2 \ll V$. In this way the Eq. (3.3) it's reduces to

$$3M_{\text{Pl}}^2 H^2 \approx V(\phi), \quad (3.14)$$

Replacing the last equation and Eq. (3.13) into Eq. (3.12), and neglecting the slow-roll parameters η and ϵ_V relative to 1 in the end. Thereby we get

$$\epsilon \approx \epsilon_V \left(1 + \frac{\beta_m^2 M^2 M_{\text{Pl}}^4 - 4MM_{\text{Pl}}^2 \beta_m f_{,\phi} V + 4f_{,\phi}^2 V^2}{4\beta_A M^2 M_{\text{Pl}}^4 + 4M_{\text{Pl}}^2 (2\beta_G + \beta_m M f_{,\phi}) V - 4f_{,\phi}^2 V^2} \right), \quad (3.15)$$

where in the last step, we have considered that $|\beta_m| \ll 1$. Additionally, making $\beta_m = 0$ and $f(\phi) = 0$ in Eq. (3.15), we have $\epsilon \approx \epsilon_V$ which corresponds to standard slow-roll inflation. We see that, the insertion of a mixing between $\dot{\phi}$ and A_0 in our model could have important consequences in the evolution of the inflationary parameters, e.g., $\dot{\phi}$, A_0 , the number of e -foldings N , among others.

In order to verify if the above considerations are correct, we will solve numerically the general background equations obtained before. To do this, we replace Eq. (3.7) (and its temporal derivative) in Eqs. (3.3) and (3.5), and in this way the Eqs. (3.3) and (3.5) are in terms of the scalar field ϕ and H . Moreover, taking into account appropriated choices for the scalar potential $V(\phi)$ and the coupling function $f(\phi)$, for example

$$V(\phi) = \frac{M^2 M_{\text{Pl}}^2}{2\alpha_c^2} \left(1 - e^{-\alpha_c \phi / M_{\text{Pl}}} \right)^2, \quad (3.16)$$

where α_c is a positive constant. This potential corresponds to the so-called α -attractor model [28]. And, for the coupling function $f(\phi)$, in this work we will consider two choices:

$$f(\phi) = \frac{\omega}{\phi^n} \quad \text{and} \quad f(\phi) = \xi e^{-\lambda \phi / M_{\text{Pl}}}, \quad (3.17)$$

where in first coupling function, ω is an arbitrary parameter with dimension of M^n ($n \in \mathbb{Z}$) and it is type power-law coupling function, which has been widely studied in the literature (see [29]). In the second one, ξ and λ are arbitrary dimensionless parameters. This type of exponential coupling function has been used

in the literature too (see [30] and [31]). Using the expressions given by Eq. (3.16) and the first coupling function in Eq. (3.17), appropriated choices for the dimensionless coupling constants $\beta_m, \beta_A, \beta_G$ and along with appropriated initial conditions, we can check numerically if the proposed model is viable to describes inflation.

In Fig. 1 we can see the evolution of $\dot{\phi}$ and A_0 during inflation and reheating, and from it, is clear that $\dot{\phi}$ and A_0 display a slow evolution during inflation, which is a fundamental condition that must fulfill the fields in the inflationary regime. Also, we can see in Fig. 2 that the ratio $A_0/\dot{\phi}$ obtained from Eq. (3.7), is nearly constant during inflation (i.e. for $Mt \lesssim 130$), since H and $f_{,\phi}$ are nearly constant during this stage. Moreover, in Fig. 1 we can see that the fields decay faster during reheating, but the ratio $A_0/\dot{\phi}$ is not nearly constant (see Fig. 2).

In Fig. 3 we have plotted the evolution of the number of e -foldings N and we see that for appropriate values of the model parameters and initial conditions used here, it is possible to obtain a suitable amount of e -foldings ($N \gtrsim 60$) for sufficient inflation. In this case, $N \approx 63$ (for $\omega = 1 M^4$) at the end of inflation. In general, inflation end when the slow-roll parameters leave of be much smaller than 1 and in our case this happens from $Mt \approx 130$ (see Fig. 7).

Using the second coupling function, in Fig. 4, we can see a similar behavior during inflation to the shown in Fig. 1, but during reheating the fields oscillate and the amplitude of A_0 decreases faster than the amplitude of $\dot{\phi}$. Furthermore, we can see in Fig. 5 that the ratio $A_0/\dot{\phi}$ during inflation is not nearly constant, but its variation is small in this phase. Additionally, in Fig. 6 is clear that for this second choice of the coupling function, we get a higher amount of e -foldings than obtained using the first coupling function. In this case, $N \approx 64$ (for $\xi = 4.5$) at the end of inflation. These results it can explain taking into account that in both cases A_0 decay faster than $\dot{\phi}$ during inflation (see Figs. 1 and 4). Therefore, from the above numerical analysis, we can say that the introduction of the coupling function $f(\phi)$ in our model of inflation, allows us to reach a suitable amount of e -foldings N for sufficient inflation. This result in very important, since without the coupling function contribution, the amount of e -foldings is smaller at the end of inflation. This result can be seen in Fig. 1 of [23]. Finally, in Fig. 7 we plotted the evolution of the new slow-roll parameters δ_1 and δ_2 which are defined by Eq. (3.11), is clear that this parameters satisfy the conditions $|\delta_1| \ll 1$ and $|\delta_2| \ll 1$ during inflation. If we consider the first coupling function given by Eq. (3.17), a similar behavior is observed.

Now, in order to verify whether the numerical inflationary solution obtained above is an attractor solution or not, we carry out an analysis on the phase-space. In this ways, in Fig 8 we show the phase-space diagram with various initial conditions and we see that the phase-space flow converge in distinct points depending on initial conditions. This observed pattern is related to the form of the potential, since it is approximately constant during inflation (flat potential) and it preserves its form under the translation $\phi \rightarrow \phi + \text{const}$. Therefore, we can say that the solution used here is independent of initial conditions (i.e. an attractor solution).

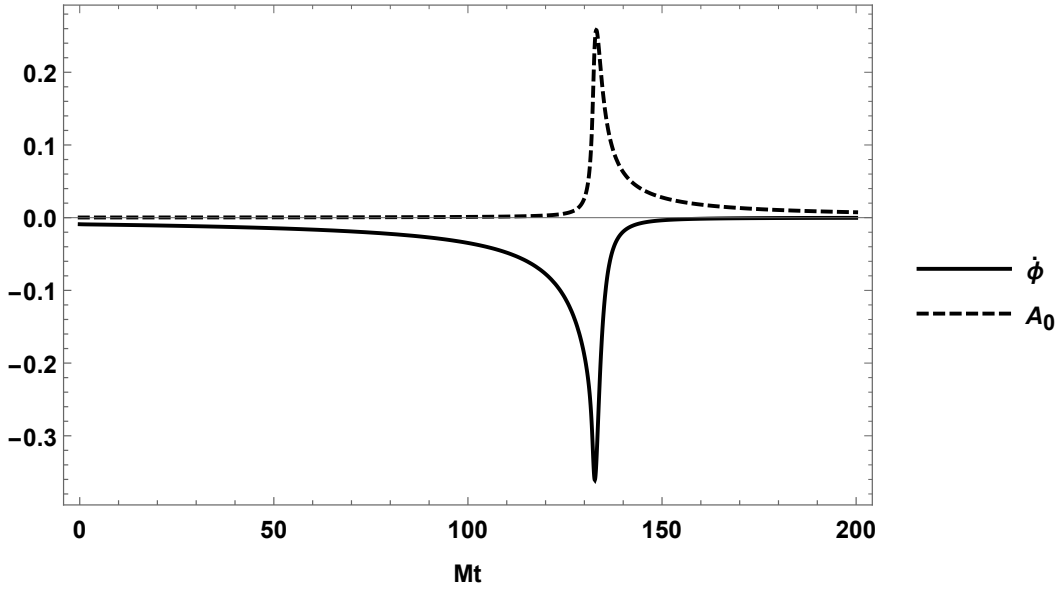


Figure 1: Evolution of $\dot{\phi}$ and A_0 (normalized by MM_{Pl} and M_{Pl} , respectively) during inflation and reheating. For this numerical simulation, we have used the potential given by Eq. (3.16) with $\alpha_c = \sqrt{6}/3$ (i.e., standard Starobinsky inflation) and the first coupling function given by Eq. (3.17) with $n = 4$, $\omega = 1 M^4$. Moreover, $\beta_m = 0.08$, $\beta_A = 1$, $\beta_G = 0.3$ and the initial conditions $\phi(0) = 5.5 M_{\text{Pl}}$, $\dot{\phi}(0) = -9 \times 10^{-3} MM_{\text{Pl}}$ and $H(0) = 0.494407 M$, which is consistent with Eqs. (3.3) and (3.7).

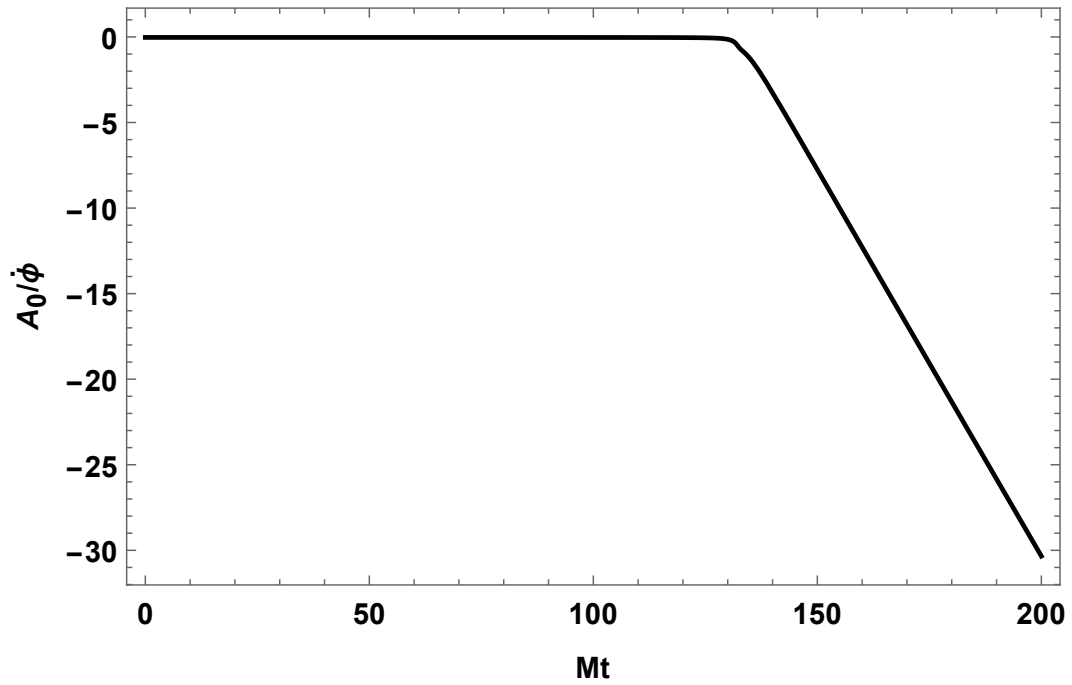


Figure 2: Evolution of $A_0/\dot{\phi}$ using the potential and the first coupling function given by Eqs. (3.16) and (3.17). In this numerical simulation the values of model parameters and initial conditions are the same as those used in Fig. 1.

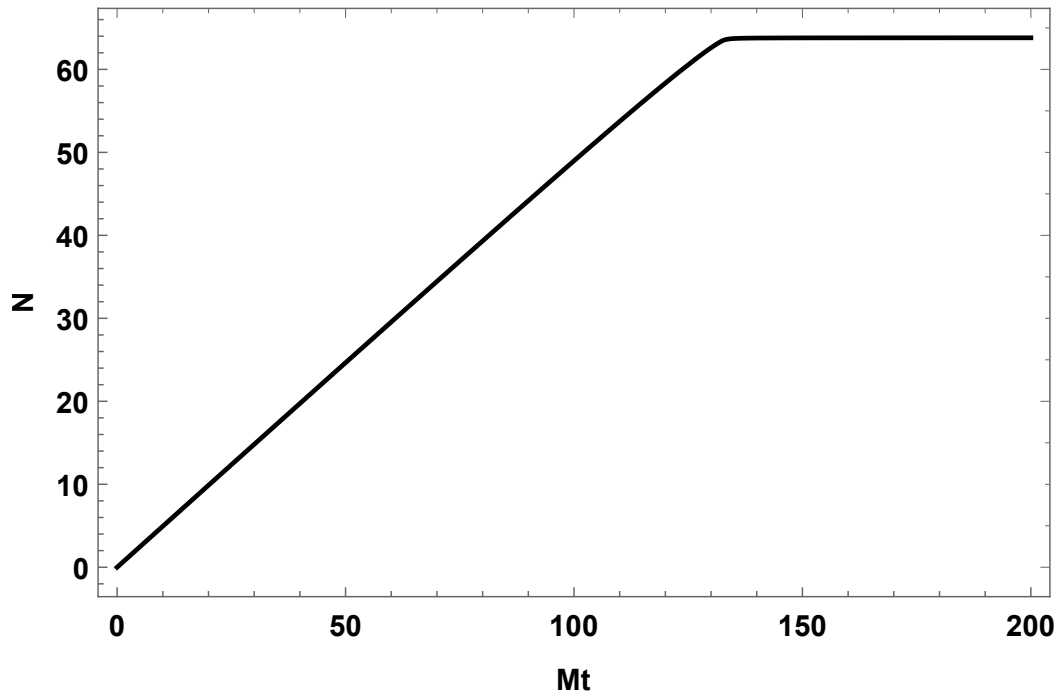


Figure 3: Evolution of the number of e -foldings N using the potential and the first coupling function given by Eqs. (3.16) and (3.17). In this numerical simulation the values of model parameters and initial conditions are the same as those used in Fig. 1.

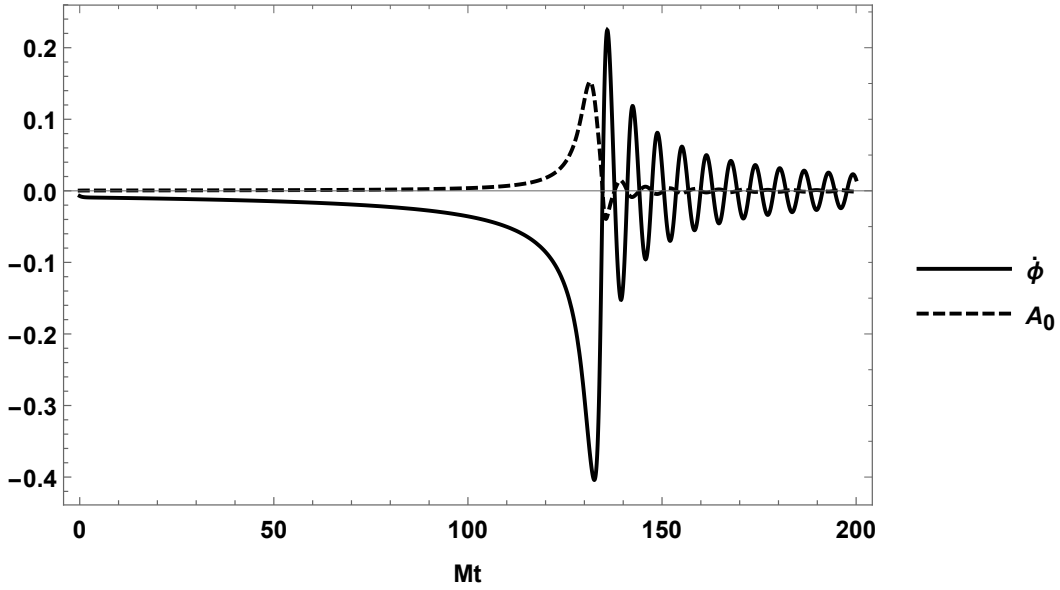


Figure 4: Evolution of $\dot{\phi}$ and A_0 (normalized by MM_{Pl} and M_{Pl} , respectively) during inflation and reheating. For this numerical simulation, we have used the potential given by Eq. (3.16) with $\alpha_c = \sqrt{6}/3$ (i.e., standard Starobinsky inflation) and the second coupling function given by Eq. (3.17) with $\xi = 4.5$, $\lambda = 1$. Additionally, $\beta_m = 0.08$, $\beta_A = 1$, $\beta_G = 0.01$ and the initial conditions $\phi(0) = 5.5 M_{\text{Pl}}$, $\dot{\phi}(0) = -9 \times 10^{-3} MM_{\text{Pl}}$ and $H(0) = 0.132553 M$, which is consistent with Eqs. (3.3) and (3.7).

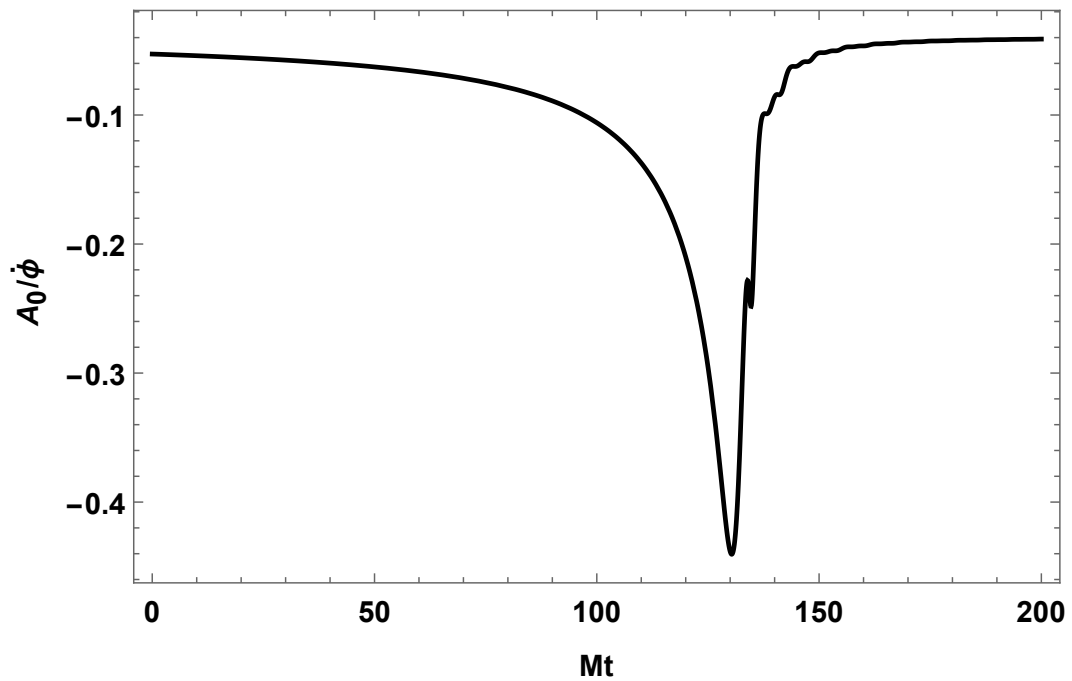


Figure 5: Evolution of $A_0/\dot{\phi}$ for the potential and the second coupling function given by Eqs. (3.16) and (3.17). In this numerical simulation the values of model parameters and initial conditions are the same as those used in Fig. 4.

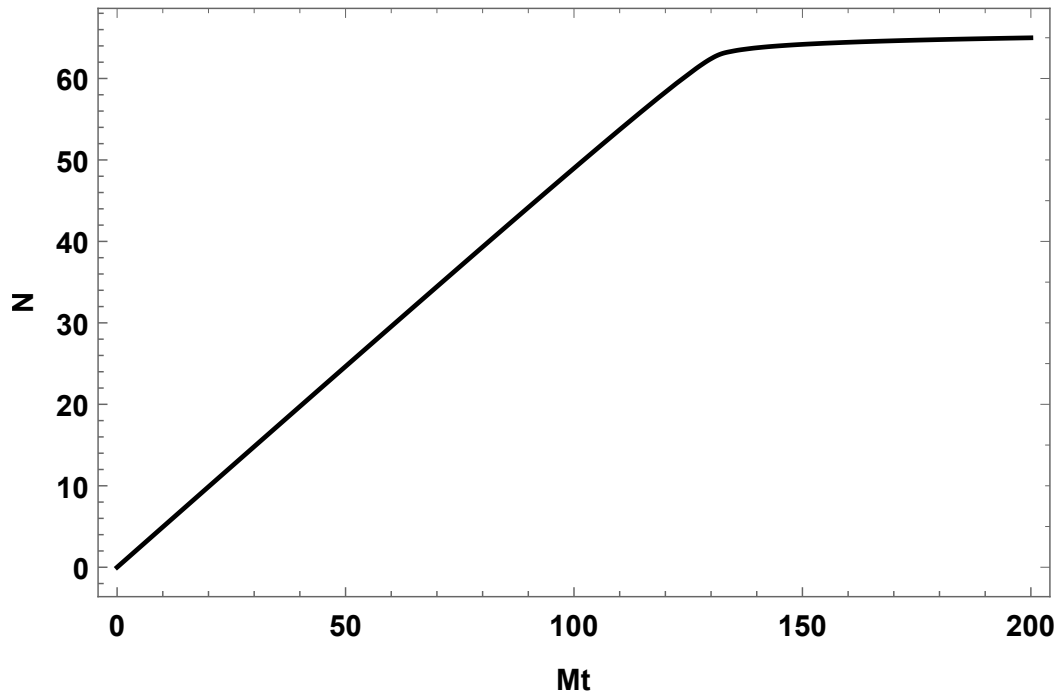


Figure 6: Evolution of the number of e -foldings N for the potential and the second coupling function given by Eqs. (3.16) and (3.17). In this numerical simulation the values of model parameters and initial conditions are the same as those used in Fig. 4.

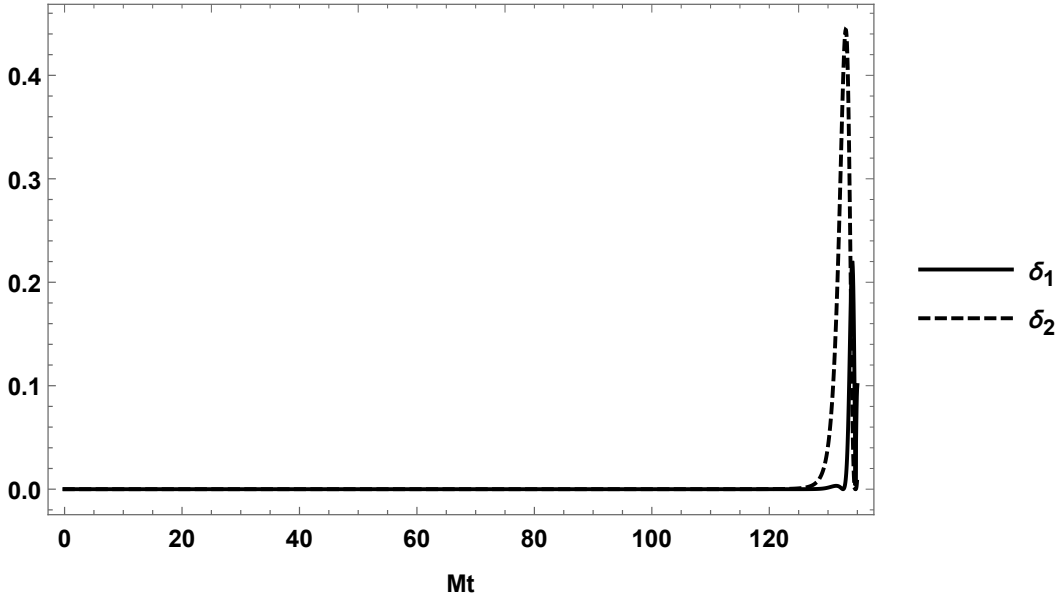


Figure 7: Evolution of slow-roll parameters δ_1 and δ_2 during inflation. In this numerical simulation we have used the potential, the coupling function, the values of model parameters and initial conditions as those used in Fig. 4.

4 No-ghosts and stability conditions

In this section, we exhibit the no-ghosts and stability conditions that our model during inflation must satisfy, i.e., absence of ghosts and Laplacian instabilities of linear cosmological perturbations. Since the general expressions associated to this analysis are very complicated, in our case, these conditions will be verified numerically taking into account particular choices for the model parameters, the potential $V(\phi)$, the coupling functions $f(\phi)$ and the initial conditions used in the previous section. For this task, we will use the general expressions for tensor, vector and scalar perturbations obtained in [23]. In this way and in order to have a self-contained document, in the next sections we reproduce literally the more important results associated with the general perturbation analysis which was performed in the sections III, IV and V of [23], and then we apply this general formalism to the present model.

4.1 Tensor perturbations

To start this analysis, it is considered the linearly perturbed line element of intrinsic tensor modes, which is given by

$$ds_t^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \quad (4.1)$$

where the tensor perturbation h_{ij} obeys the usual transverse and traceless conditions $\nabla^j h_{ij} = 0$ and $h_i^i = 0$. Expanding the action (2.1) up to quadratic order in h_{ij} and integrating it by parts, the second-order action

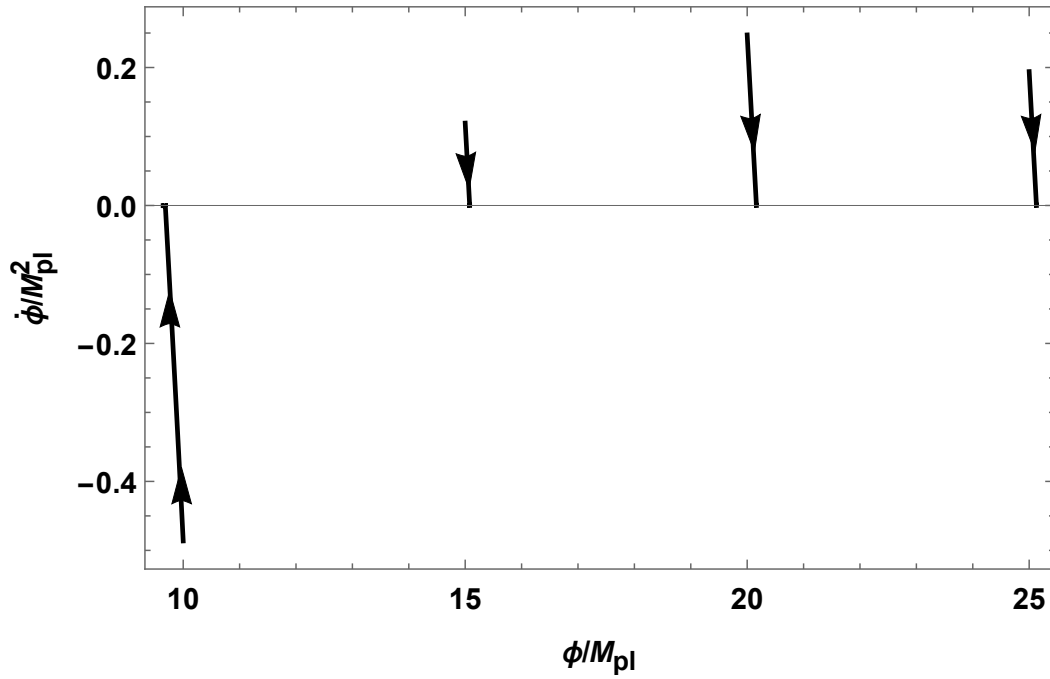


Figure 8: Phase-space structure for the inflationary solution. In this numerical simulation we have used the potential, the coupling function, the values of model parameters as those used in Fig. 4, and various initial conditions for ϕ were used.

of tensor perturbations yields

$$S_t^{(2)} = \int dt d^3x \frac{a^3 q_t}{8} \delta^{ik} \delta^{jl} \left[\dot{h}_{ij} \dot{h}_{kl} - \frac{c_t^2}{a^2} (\partial h_{ij})(\partial h_{kl}) \right], \quad (4.2)$$

where the symbol ∂ represents the spatial partial derivative and q_t and c_t^2 are given by Eqs. (3.3) and (3.4) of [23]. For the model under consideration, the associated quantities for tensor perturbations q_t and c_t^2 , are given by

$$q_t = M_{\text{Pl}}^2 - \frac{(\beta_m M - 6f_{,\phi} H^2)(M\beta_G \beta_m + 2f_{,\phi}(\beta_A M^2 + 3\beta_G H^2))\dot{\phi}^2}{4(\beta_A M^2 + 6\beta_G H^2)^2}, \quad (4.3)$$

$$c_t^2 = 1 + \frac{2(\beta_m M - 6f_{,\phi} H^2)(M\beta_G \beta_m + 2f_{,\phi}(\beta_A M^2 + 3\beta_G H^2))\dot{\phi}^2}{4M_{\text{Pl}}^2(\beta_A M^2 + 6\beta_G H^2)^2 - (\beta_m M - 6f_{,\phi} H^2)(M\beta_G \beta_m + 2f_{,\phi}(\beta_A M^2 + 3\beta_G H^2))\dot{\phi}^2}, \quad (4.4)$$

where c_t represent the propagation speed of gravitational waves on the FLRW background and Eq. (3.7) was used to express A_0 in terms of $\dot{\phi}$. Furthermore, to guarantee the lack of ghost or Laplacian instabilities in the tensor sector, it must be fulfilled that $q_t > 0$ and $c_t^2 > 0$.

4.2 Vector perturbations

For perturbations in the vector sector, it's used the perturbed line element in the flat gauge:

$$ds_v^2 = -dt^2 + 2V_i dt dx^i + a^2(t) \delta_{ij} dx^i dx^j, \quad (4.5)$$

where V_i is the vector perturbation obeying the transverse condition $\nabla^i V_i = 0$. The temporal and spatial components of A^μ associated with the intrinsic vector sector are expressed as

$$A_0 = A_0(t), \quad A_i = Z_i(t, x^i), \quad (4.6)$$

where Z_i is the intrinsic vector perturbation which satisfies the transverse condition $\partial^i Z_i = 0$. Expanding Eq. (2.1) up to quadratic order in perturbations and using the background Eqs. (2.13) and (2.16) of [23], the resulting second-order action in the vector sector yields

$$S_v^{(2)} = \int dt d^3x \sum_{i=1}^2 \left[\frac{a q_v}{2} \dot{Z}_i^2 - \frac{1}{2a} \alpha_1 (\partial Z_i)^2 - \frac{a}{2} \alpha_2 Z_i^2 + \frac{1}{2a} \alpha_3 (\partial V_i)(\partial Z_i) + \frac{q_t}{4a} (\partial V_i)^2 \right], \quad (4.7)$$

where q_v , α_1 , α_2 and α_3 are given by Eqs. (4.4)-(4.7) of [23]. In our case, we get

$$\begin{aligned} q_v &= 1, \\ \alpha_1 &= 1, \\ \alpha_2 &= \beta_A M^2 + 4\beta_G \dot{H} + 6\beta_G H^2, \\ \alpha_3 &= -2\beta_G A_0 + \dot{\phi} f_{,\phi}, \end{aligned} \quad (4.8)$$

and q_t is defined by Eq. (4.3).

Varying the second-order action (4.7) with respect to V_i , we obtain the constraint equation:

$$\partial^2(\alpha_3 Z_i + q_t V_i) = 0, \quad (4.9)$$

from this equation it is evident that $V_i = -\alpha_3 Z_i / q_t$ (taking the integration constant as 0). Replacing this relation into Eq. (4.7), we obtain

$$S_v^{(2)} = \int dt d^3x \sum_{i=1}^2 \frac{aq_v}{2} \left[\dot{Z}_i^2 - \frac{c_v^2}{a^2} (\partial Z_i)^2 - \frac{\alpha_2}{q_v} Z_i^2 \right], \quad (4.10)$$

where

$$c_v^2 = \frac{2\alpha_1 q_t + \alpha_3^2}{2q_t q_v}, \quad (4.11)$$

and replacing the Eqs. (4.3) and (4.8), we obtain

$$c_v^2 = 1 + \frac{2M^2(\beta_G \beta_m + M\beta_A f_{,\phi})^2 \dot{\phi}^2}{4M_{\text{Pl}}^2(\beta_A M^2 + 6\beta_G H^2)^2 - (\beta_m M - 6f_{,\phi} H^2)(M\beta_G \beta_m + 2f_{,\phi}(\beta_A M^2 + 3\beta_G H^2))\dot{\phi}^2}. \quad (4.12)$$

Newly, it must be fulfilled that $q_v > 0$ and $c_v^2 > 0$ to guarantee the lack of ghost or Laplacian instabilities of vector perturbations. From Eq. (4.8), is evident that the first condition is satisfied.

4.3 Scalar perturbations

For scalar perturbations, the perturbed line element in the flat gauge:

$$ds_s^2 = -(1 + 2\alpha)dt^2 + 2\partial_i \chi dt dx^i + a^2(t) \delta_{ij} dx^i dx^j, \quad (4.13)$$

is considered, where α and χ are scalar metric perturbations. The components of the vector field it can write in the form

$$A^0 = -A_0(t) + \delta A, \quad A_i = \partial_i \psi, \quad (4.14)$$

where δA is the perturbation of the temporal vector component A^0 , and ψ is the longitudinal scalar perturbation. The scalar field ϕ is decomposed into the background and perturbed parts as

$$\phi = \phi_0(t) + \delta\phi. \quad (4.15)$$

Expanding the action (2.1) up to quadratic order in scalar perturbations α , χ , δA , ψ and $\delta\phi$. In doing so, we use the background Eqs. (2.13) and (2.16) of [23] to eliminate the terms f_2 and $f_{3,\phi}$. Then, the second-order action for scalar perturbations can be expressed as

$$S_s^{(2)} = \int dt d^3x (\mathcal{L}_s^\phi + \mathcal{L}_s^{GP}), \quad (4.16)$$

where

$$\begin{aligned} \mathcal{L}_s^\phi = & a^3 \left[D_1 \dot{\delta\phi}^2 + D_2 \frac{(\partial\delta\phi)^2}{a^2} + D_3 \delta\phi^2 + \left(D_4 \dot{\delta\phi} + D_5 \delta\phi + D_6 \frac{\partial^2 \delta\phi}{a^2} \right) \alpha - (D_6 \dot{\delta\phi} - D_7 \delta\phi) \frac{\partial^2 \chi}{a^2} \right. \\ & \left. + (D_8 \dot{\delta\phi} + D_9 \delta\phi) \delta A + D_{10} \delta\phi \frac{\partial^2 \psi}{a^2} \right], \end{aligned} \quad (4.17)$$

and

$$\begin{aligned} \mathcal{L}_s^{GP} = & a^3 \left[\left(w_1 \alpha - w_2 \frac{\delta A}{A_0} \right) \frac{\partial^2 \chi}{a^2} - w_3 \frac{(\partial \alpha)^2}{a^2} + w_4 \alpha^2 - \left(w_3 \frac{\partial^2 \delta A}{a^2 A_0} - w_8 \frac{\delta A}{A_0} + w_3 \frac{\partial^2 \dot{\psi}}{a^2 A_0} + w_6 \frac{\partial^2 \psi}{a^2} \right) \alpha \right. \\ & \left. - w_3 \frac{(\partial \delta A)^2}{4a^2 A_0^2} + w_5 \frac{\delta A^2}{A_0^2} + \{ w_3 \dot{\psi} - (w_2 - A_0 w_6) \psi \} \frac{\partial^2 \delta A}{2a^2 A_0^2} - w_3 \frac{(\partial \dot{\psi})^2}{4a^2 A_0^2} + w_7 \frac{(\partial \psi)^2}{2a^2} \right], \end{aligned} \quad (4.18)$$

where *GP* means Generalized Proca theories and the coefficients $D_{1,\dots,10}$ and $w_{1,\dots,8}$ were defined in the appendix of [23]. In our case, these coefficients are given by

$$\begin{aligned} D_1 &= \frac{1}{2}, \\ D_2 &= -\frac{1}{2}, \\ D_3 &= \frac{9}{2} H^3 A_0 f_{,\phi\phi} + \frac{1}{2} V_{,\phi\phi}, \\ D_4 &= -\dot{\phi} + 3H^2 A_0 f_{,\phi} + \frac{1}{2} \beta_m M A_0, \\ D_5 &= 3H^2 \dot{\phi} A_0 f_{,\phi\phi} - V_{,\phi}, \\ D_6 &= -2H A_0 f_{,\phi}, \\ D_7 &= -3H^2 \dot{\phi} A_0 f_{,\phi} + 2H \dot{\phi} A_0 f_{,\phi\phi} + \dot{\phi} + \frac{1}{2} \beta_m M A_0, \\ D_8 &= -\frac{2\dot{\phi} D_1 + D_4 + 3H D_6}{A_0}, \\ D_9 &= -\frac{D_5}{A_0} + 6H^2 \dot{\phi} f_{,\phi\phi} - \frac{V_{,\phi}}{A_0}, \\ D_{10} &= -2\dot{H} f_{,\phi} - 3H^2 f_{,\phi} + \frac{1}{2} \beta_m M \end{aligned} \quad (4.19)$$

and

$$\begin{aligned} w_1 &= -2H \dot{\phi} A_0 f_{,\phi} - 2H M_{\text{Pl}}^2 - 2\beta_G H A_0^2, \\ w_2 &= w_1 + 2H q_t - \dot{\phi} D_6, \\ w_3 &= -2A_0^2 q_v, \\ w_4 &= -3H^2 M_{\text{Pl}}^2 + 3H^2 \beta_G A_0^2 + \frac{3}{2} H^2 \dot{\phi} A_0 f_{,\phi} + \frac{1}{2} \dot{\phi}^2 - \frac{3}{4} \beta_m M \dot{\phi} A_0, \\ w_5 &= w_4 - \frac{3H(w_1 + w_2)}{2} - \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \beta_m M \dot{\phi} A_0, \\ w_6 &= -\frac{2w_1 - w_2 - 2\dot{\phi} D_6 + 4H M_{\text{Pl}}^2 + 4\beta_G H A_0^2}{A_0}, \\ w_7 &= -4\beta_G \dot{H} - \frac{3H^2 \dot{\phi} f_{,\phi}}{A_0} + \frac{\beta_m M \dot{\phi}}{2A_0}, \\ w_8 &= 3H w_1 - 2w_4 + \dot{\phi} D_4. \end{aligned} \quad (4.20)$$

From Eqs. (4.17) and (4.18) it is clear that the fields α , χ and δA are nondynamical. Therefore, varying the action (4.16) with respect to α , χ and δA , we obtain the three constraint equations in Fourier space:

$$D_4 \dot{\delta\phi} + D_5 \delta\phi + 2w_4 \alpha + w_8 \frac{\delta A}{A_0} + \frac{k^2}{a^2} \left(w_3 \frac{\dot{\psi}}{A_0} + w_6 \psi - D_6 \delta\phi - 2w_3 \alpha - w_1 \chi + w_3 \frac{\delta A}{A_0} \right) = 0, \quad (4.21)$$

$$D_6\dot{\delta\phi} - D_7\delta\phi - w_1\alpha + w_2\frac{\delta A}{A_0} = 0, \quad (4.22)$$

$$D_8\dot{\delta\phi} + D_9\delta\phi + w_8\frac{\alpha}{A_0} + 2w_5\frac{\delta A}{A_0^2} - \frac{k^2}{a^2}\frac{1}{A_0}\left(\frac{w_3}{2}\frac{\dot{\psi}}{A_0} + \frac{A_0w_6 - w_2}{2}\frac{\psi}{A_0} - w_3\alpha - w_2\chi + \frac{w_3}{2}\frac{\delta A}{A_0}\right) = 0, \quad (4.23)$$

we solve Eqs. (4.21)-(4.23) for α , χ and δA , and eliminate these variables from the action (4.16). Then, the second-order action of scalar perturbations can be expressed in the form

$$S_v^{(2)} = \int dt d^3x a^3 \left(\dot{\vec{\chi}}^t \mathbf{K} \dot{\vec{\chi}} - \frac{k^2}{a^2} \vec{\chi}^t \mathbf{G} \vec{\chi} - \vec{\chi}^t \mathbf{M} \vec{\chi} - \vec{\chi}^t \mathbf{B} \dot{\vec{\chi}} \right), \quad (4.24)$$

where \mathbf{K} , \mathbf{G} , \mathbf{M} and \mathbf{B} are 2×2 matrices, and $\vec{\chi}$ is defined by

$$\vec{\chi}^t = (\psi, \delta\phi). \quad (4.25)$$

In the small-scale limit, the leading-order contributions to the matrix \mathbf{M} do not contain the k^2 terms. We shift the k^2 terms appearing in \mathbf{B} to the matrix components of \mathbf{G} after integrating them by parts. For $k \rightarrow \infty$, the components of \mathbf{K} and \mathbf{G} are given, respectively, by

$$\begin{aligned} K_{11} &= \frac{w_1^2 w_5 + w_2^2 w_4 + w_1 w_2 w_8}{A_0^2 (w_1 - 2w_2)^2}, \\ K_{22} &= D_1 + \frac{D_6}{w_1 - 2w_2} \left(D_4 + \frac{w_4 + 4w_5 + 2w_8}{w_1 - 2w_2} D_6 + 2A_0 D_8 \right), \\ K_{12} = K_{21} &= -\frac{1}{2A_0 (w_1 - 2w_2)} \left[w_2 D_4 + \frac{w_1 (4w_5 + w_8) + 2w_2 (w_4 + w_8)}{w_1 - 2w_2} D_6 + A_0 w_1 D_8 \right], \end{aligned} \quad (4.26)$$

and

$$\begin{aligned} G_{11} &= \dot{E}_1 + H E_1 - \frac{4A_0^2}{w_3} E_1^2 - \frac{w_7}{2}, \\ G_{22} &= \dot{E}_2 + H E_2 - \frac{2A_0}{w_2} D_7 E_3 - \frac{4A_0^2}{w_3} E_3^2 - D_2, \\ G_{12} = G_{21} &= \dot{E}_3 + H E_3 - \frac{4A_0^2}{w_3} E_1 E_3 + \frac{w_2}{2A_0 (w_1 - 2w_2)} D_7 + \frac{D_{10}}{2}, \end{aligned} \quad (4.27)$$

where E_1 , E_2 and E_3 are defined as

$$\begin{aligned} E_1 &= \frac{w_6}{4A_0} - \frac{w_1 w_2}{4A_0^2 (w_1 - 2w_2)}, \\ E_2 &= -\frac{D_6^2}{2(w_1 - 2w_2)}, \\ E_3 &= \frac{w_2 D_6}{2A_0 (w_1 - 2w_2)}. \end{aligned} \quad (4.28)$$

In our case, the explicit form for the components Eqs. (4.26) and (4.27) are obtained replacing $D_{1,\dots,10}$ and $w_{1,\dots,8}$ given by Eqs. (4.19) and (4.20), but the resulting expressions are too long to be written here.

In order to ensure the absence of scalar ghosts, the kinetic matrix \mathbf{K} must be positive definite. In other words, the determinants of principal submatrices of \mathbf{K} need to be positive. Thus, we require the following two no-ghost conditions:

$$K_{11} > 0 \quad \text{or} \quad K_{22} > 0, \quad (4.29)$$

$$q_s \equiv K_{11}K_{22} - K_{12}^2. \quad (4.30)$$

In the small-scale limit, the dispersion relation following from the action (4.24) with frequency ω is given by

$$\det \left(\omega^2 \mathbf{K} - \frac{k^2}{a^2} \mathbf{G} \right) = 0, \quad (4.31)$$

Introducing the scalar sound speed c_s as $\omega^2 = c_s^2 k^2 / a^2$, the above dispersion relation leads to the two scalar propagation speed squares:

$$c_{s1}^2 = \frac{K_{11}G_{22} + K_{22}G_{11} - 2K_{12}G_{12} + \sqrt{(K_{11}G_{22} + K_{22}G_{11} - 2K_{12}G_{12})^2 - 4(K_{11}K_{22} - K_{12}^2)(G_{11}G_{22} - G_{12}^2)}}{2(K_{11}K_{22} - K_{12}^2)}, \quad (4.32)$$

$$c_{s2}^2 = \frac{K_{11}G_{22} + K_{22}G_{11} - 2K_{12}G_{12} - \sqrt{(K_{11}G_{22} + K_{22}G_{11} - 2K_{12}G_{12})^2 - 4(K_{11}K_{22} - K_{12}^2)(G_{11}G_{22} - G_{12}^2)}}{2(K_{11}K_{22} - K_{12}^2)}. \quad (4.33)$$

For the absence of Laplacian instabilities, we require that

$$c_{s1}^2 > 0, \quad c_{s2}^2 > 0. \quad (4.34)$$

In order for the system to be completely stable against ghosts and Laplacian instabilities, all the dynamical degrees of freedom must simultaneously satisfy both of the conditions of no-ghosts and positive squared sound speeds, however the expressions obtained here for q_t , c_t^2 , q_v , c_v^2 , K_{22} , q_s , c_{s1}^2 , c_{s2}^2 are in terms of complicated combinations, which include several model parameters and other quantities as H , $f_{,\phi}$ and the field ϕ and its derivative, then is not easy to write explicit no-ghosts and stability conditions. Therefore, in this work we will verify numerically these conditions taking into account particular choices for the model parameters, the potential $V(\phi)$, the coupling functions $f(\phi)$ and the initial conditions used in the previous section. Thereby, in Figs. 9, 10, 11, 12, 13 and 14 we can see that for our model the no-ghosts and stability conditions during inflation are satisfied, i.e. there are no ghosts and Laplacian instabilities of tensor, vector, and scalar perturbations. In Figs. 10 and 13 we observe that the given quantities are nearly 1 during inflation (in this work, we have used the natural units in which the speed of light c is equivalent to 1) and in Fig. 11 and 14 we see that K_{22} is about 1/2 during inflation. However, at the end of inflation these figures show a large deviation which is induced by the time variation of ϕ (besides, in Fig. 1 we can see that $\dot{\phi}$ reaches a minimum around the end of inflation). This same behavior is observed in Fig. 4 too. Moreover, in this context, the general condition is that these quantities must be positive to guarantee the absence of instabilities within the model, since in this regimen (inflation), until now, there are no constraints imposed by the observations in early times, like those imposed by the gravitational waves in late times, $z < 0.09$, (the speed of gravitational waves c_t is the same as the speed of light, which places tight constraints on dark energy models constructed in the framework of modified gravitational theories). In this sense, this restriction does not apply to high redshift values (inflation), thus, the model (scalar-vector-tensor theories) under consideration in this work

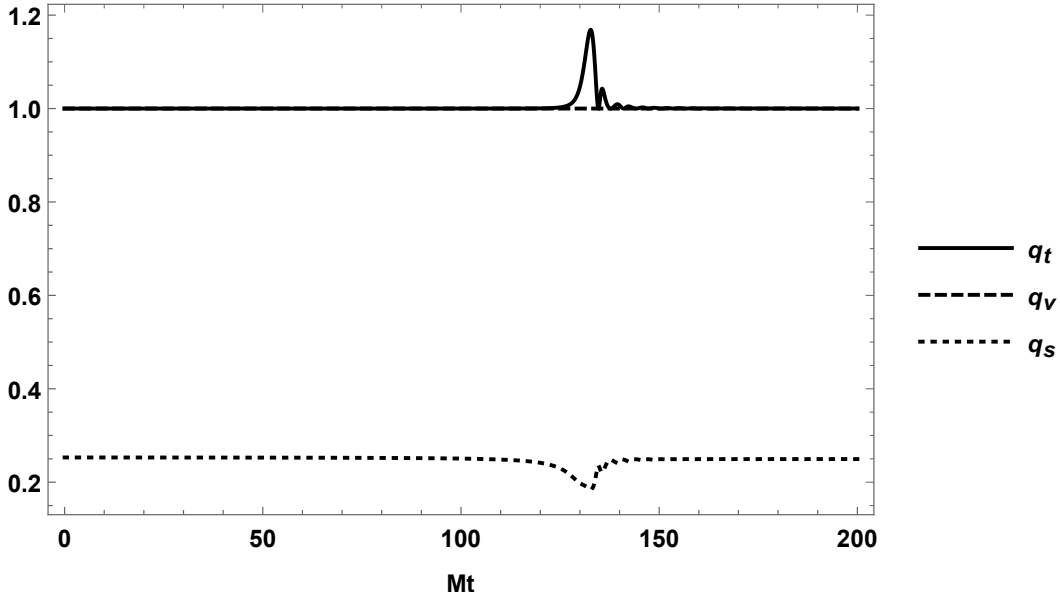


Figure 9: Evolution of q_t , q_v and q_s during inflation. In this numerical simulation we have used the potential, the second coupling function, the values of model parameters and initial conditions as those used in Fig. 4

could be used in an inflationary context. Finally, from the above, we can say that using the second coupling function we obtain better results than those obtained using the first coupling function (since in Fig. 13 the exponential growth of c_v^2 and c_{s1}^2 after inflation could be problematic).

5 Conclusions

In this work, we have studied inflation in a particular scalar-vector-tensor theory of gravitation without the $U(1)$ gauge symmetry. This model was constructed from the more general action given by Eqs. (2.1) and (2.2) using certain specific choices for the Lagrangians and the coupling functions (see Eq. (2.4)). From this model, we obtained the general background equations of motion on the flat FLRW spacetime and using concrete choices for the potential (see Eq. (3.16)), the coupling functions (see Eq. (3.17)), suitable dimensionless coupling constants and initial conditions, was possible to verify numerically that this model of inflation is viable. In this way, in Fig. 1 we plotted the evolution of $\dot{\phi}$ and A_0 during inflation and reheating, and from it, is clear that $\dot{\phi}$ and A_0 display a slow evolution during inflation. Further, the fields decay faster during reheating. In Fig. 3 the evolution of the number of e -foldings N was plotted and we see that in this case, it is possible to obtain a suitable amount of e -foldings ($N \gtrsim 60$) for sufficient inflation. Moreover, $N \approx 63$ (for $\omega = 1 M^4$) at the end of inflation. Using the second coupling function, in Fig. 4, we can see a similar behavior during inflation to the shown in Fig. 1, but during reheating the fields oscillate and the amplitude of A_0 decreases faster than the amplitude of $\dot{\phi}$. Furthermore, in Fig. 6 is clear that for the second choice of

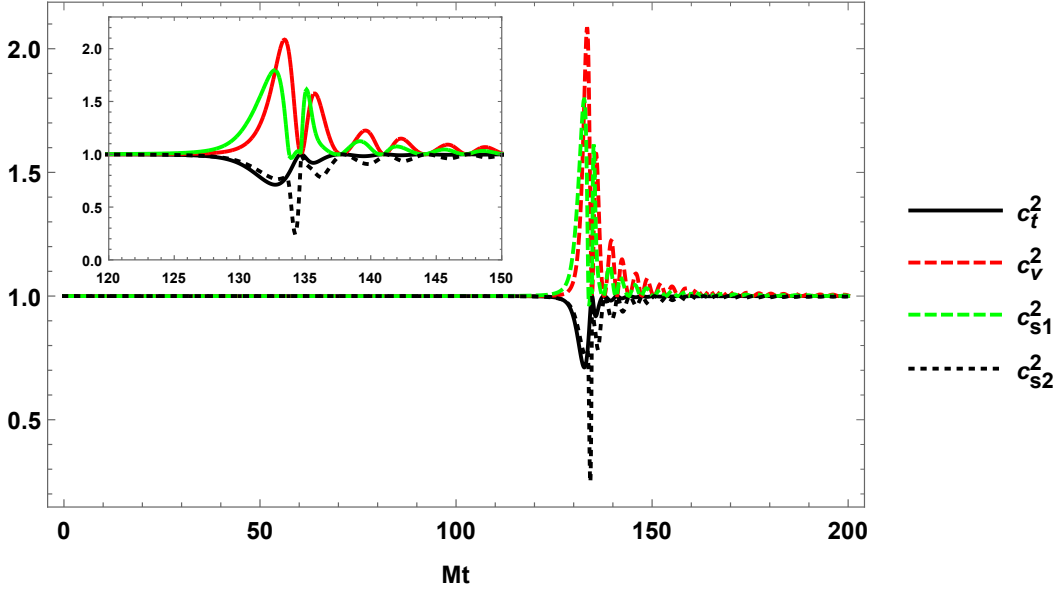


Figure 10: Evolution of c_t^2 , c_v^2 , $c_{s_1}^2$ and $c_{s_2}^2$ during inflation. In this numerical simulation we have used the potential, the second coupling function, the values of model parameters and initial conditions as those used in Fig. 4

the coupling function (see Eq. (3.17)), we got a similar amount of e -foldings as that obtained using the first coupling function. In this case, $N \approx 64$ (for $\xi = 4.5$) at the end of inflation. Therefore, we can say that the introduction of the coupling function $f(\phi)$ in our model of inflation, allows us to reach a suitable amount of e -foldings N for sufficient inflation. This is a remarkable result, since without the coupling function contribution, the amount of e -foldings is smaller at the end of inflation, as has been demonstrated in [23]. In Fig. 7 we plotted the evolution of the new slow-roll parameters δ_1 and δ_2 which are defined by Eq. (3.11) and from it, is evident that the slow-roll parameters satisfy the conditions $|\delta_1| \ll 1$ and $|\delta_2| \ll 1$ during inflation. Also, we have shown that the numerical inflationary solution obtained here is an attractor solution (see Fig 8). Finally, the no-ghosts and stability conditions that the model during inflation must satisfy, i.e., absence of ghosts and Laplacian instabilities of linear cosmological perturbations were obtained. For do this, the general expressions for tensor, vector and scalar perturbations obtained in [23] were used, reproducing literally the main results associated with the general stability analysis which was performed in the sections III, IV and V of [23], and then we applied this general formalism to our model. In this sense, the main expressions which must be positive to guarantee absence of ghosts and Laplacian instabilities were obtained, and they are: q_t , c_t^2 , q_v , c_v^2 , K_{22} , q_s , $c_{s_1}^2$, $c_{s_2}^2$ (see Eqs. (4.3), (4.4), (4.8), (4.12), (4.29), (4.30), (4.32) and (4.33)). Additionally, the positivity condition which these quantities must be satisfied were verified numerically too (see Figs. 9-14).

We must emphasize that the main goal of this work was devoted to the demonstration of a viable model of

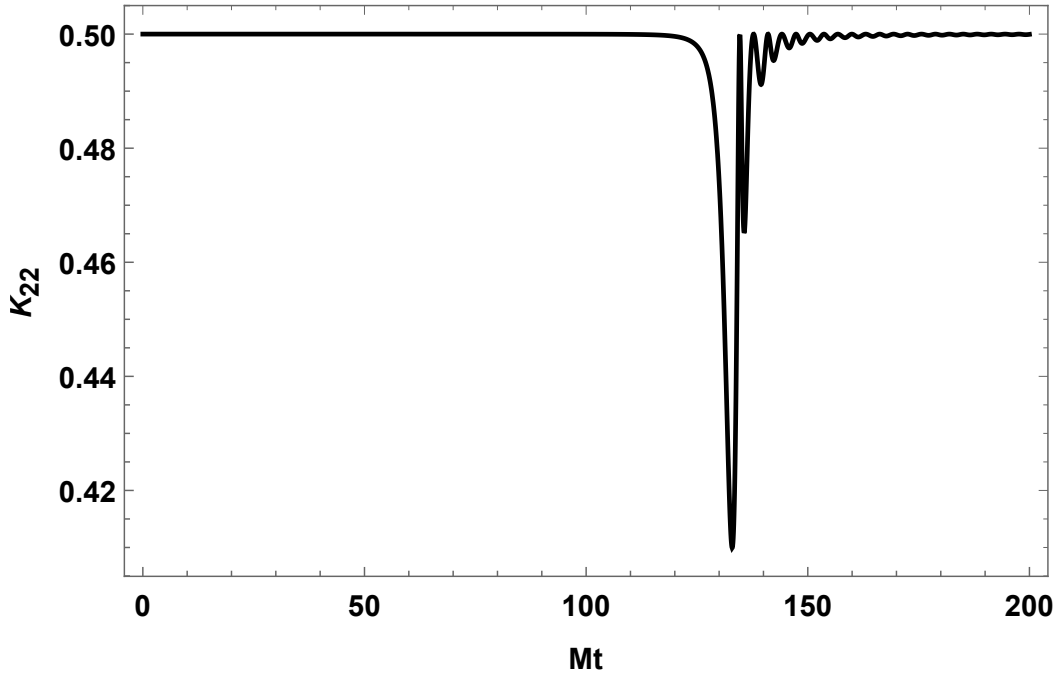


Figure 11: Evolution of the component K_{22} during inflation. In this numerical simulation we have used the potential, the second coupling function, the values of model parameters and initial conditions as those used in Fig. 4

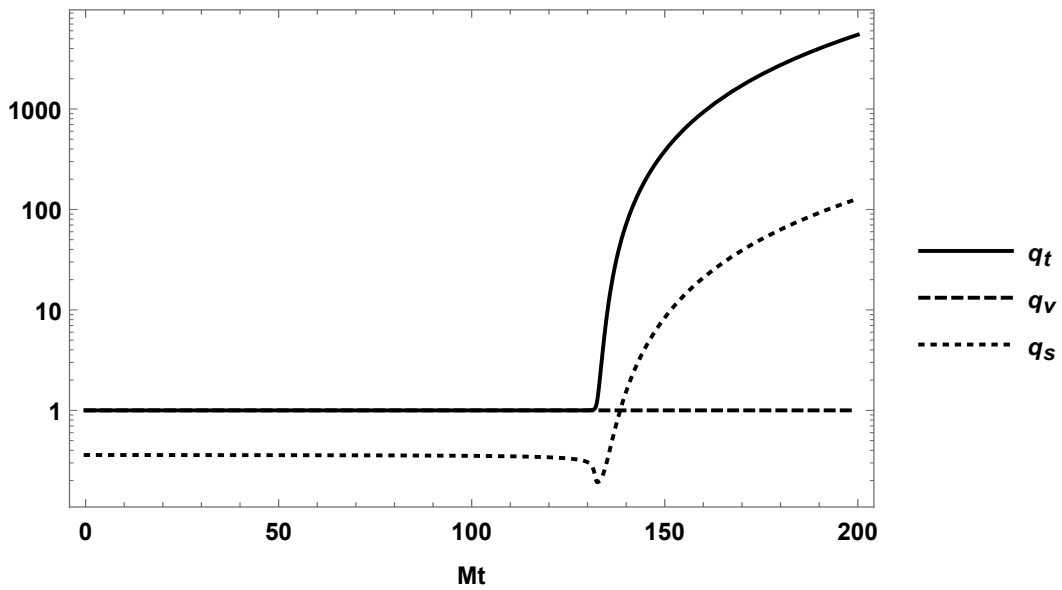


Figure 12: Evolution of q_t , q_v and q_s during inflation. In this numerical simulation we have used the potential, the first coupling function, the values of model parameters and initial conditions as those used in Fig. 1

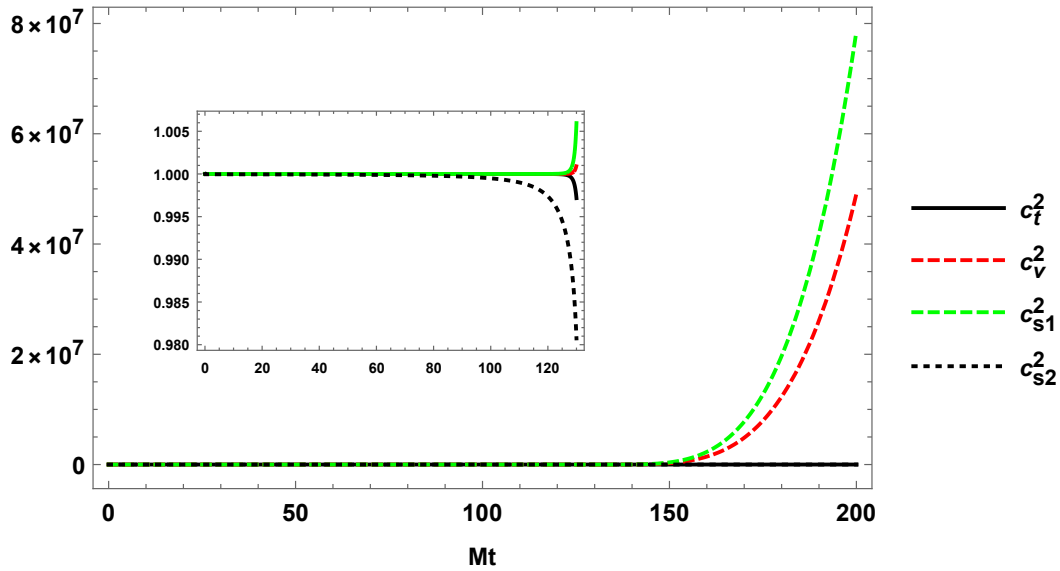


Figure 13: Evolution of c_t^2 , c_v^2 , c_{s1}^2 and c_{s2}^2 during inflation. In this numerical simulation we have used the potential, the first coupling function, the values of model parameters and initial conditions as those used in Fig. 1

inflation that was healthy at the theoretical level, which was successfully achieved. Phenomenological consequences of this pathology-free model is therefore of interest for further studies. The observational bounds on the scalar spectral index and the tensor-to-scalar ratio can constrain/falsify this model, but that kind of analysis is beyond the scope of this work and could be addressed later.

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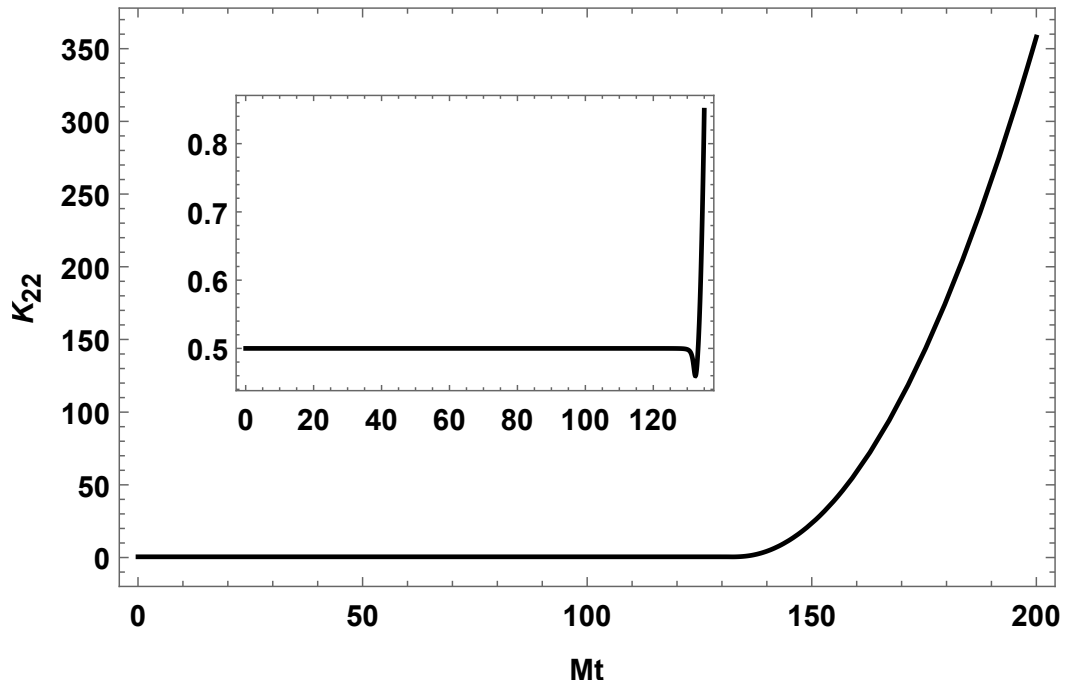


Figure 14: Evolution of the component K_{22} during inflation. In this numerical simulation we have used the potential, the first coupling function, the values of model parameters and initial conditions as those used in Fig. 1

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