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A NOTE ON SEMI-OPEN SETS IN TRICLOSURE SPACES

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Abstract. In this paper, we used the notion of triclosure spaces to introduce and study the concept of semi-open set in triclosure spaces. Besides, we show some of their properties. Moreover, the notions of semi-continuous and semi-irresolute functions in a triclosure spaces are studied. Furthermore, we prove some of their properties.

Keywords: Triclosure spaces; semi-open sets; semi-continuous functions; semi-irresolute functions.

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1. Introduction

Levine in 1963 [9] introduced the notions of semi-open sets and semi-continuity in a topological space (X,τ). On the other hand, the concept of a tritopological space was introduced by Kovar in 2000 [8]. Moreover, the idea of closure space was introduced by Cech in 1968 [3] and then has been studied by many mathematicians, see [1] and [4]. Otherwise, a function $u : P(X) \to P(X)$ defined on the power set $P(X)$ of a set X is called a closure oeprator on X and the pair (X,u) is called a closure space if the following axioms are satisfied: (1) $u\emptyset = \emptyset$,(2) $A \subseteq uA$ for every $A \subseteq X$ and (3) if $A \subseteq B$, then $uA \subseteq uB$ for all $A,B \in X$. After that, the notion of biclosure space was introduced by Boonkop and Khampakdee in 2008 [2] and then it has been

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studied by many authors in this field, see [1], [5] and [7]. On the other hand, taking into account those notions, Granados in 2020 [6] introduced and studied new notions on triclosure spaces. A triclosure space is a triplet (X,u_1,u_2,u_3) where u_1,u_2 and u_3 are three closure operators on X, besides a subset A of (X,u_1,u_2,u_3) is closed if $u_1u_2u_3A = A$. The complement of a closed set is called open set. In this paper, we took the idea of triclosure space and we introduce the concept of semi-open set in a triclosure space. Furthermore, some of their properties are studied. Besides, the notions of semi-open, open, semi-continuous and semi-irresolute functions are introduced, we also study some of their properties.

2. Semi-Open Sets in Triclosure Spaces

In this section, we introduce the concept of semi-open set in triclosure space. Besides, we study some of their properties. Throughout this section, the $^\circ$ means the interior of any set.

Definition 2.1. Let (X,u_1,u_2,u_3) be a triclosure space and $A \subseteq X$. Then, A is said to be semi-open if there exits an open set V in (X,u_1) such that $V \subseteq A \subseteq u_2u_3V$. The complement of a semi-open set is called semi-closed.

Remark 2.1. By the Definition 2.1, we can find the semi-open sets, as well as, $V \subseteq u_2u_3V^\circ$.

Remark 2.2. If A is open (respectively, closed) in (X,u_1), then A is semi-open (respectively, semi-closed) in (X,u_1,u_2,u_3). The converse need not be true as can be seen in the following example.

Example 2.1. Let $X = \{a,b,c,d\}$ and define a closure operator u_1 on X as $u_1\emptyset = \emptyset$, $u_1X = X = u_1\{a,b\}$, $u_1\{a\} = \{a,c,d\}$, $u_1\{b\} = \{b,c,d\}$, $u_1\{c,d\} = \{c,d\}$. Define the closure operator u_2 on X as $u_2\emptyset = \emptyset$, $u_2X = X = u_2\{a,c,d\} = u_2\{c\}$ and define the closure operator u_3 on X as $u_3\emptyset = \emptyset$, $u_3\{a,c,d\} = \{a,c,d\}$, $u_3\{c\} = \{c\}$ and u_3 in the rest of sets from X is X. Then, $\{a,b,c\}$ is semi-open in (X,u_1,u_2,u_3) but $\{a,b,c\}$ is not open in (X,u_1), (X,u_2) and (X,u_3). Furthermore, $\{d\}$ is semi-closed, but it is not closed in (X,u_1), (X,u_2) and (X,u_3).

Theorem 2.1. *Let (X,u_1,u_2,u_3) be a triclosure space and $A \subseteq X$. Then, A is semi-closed if and only if there exits a closed subset B of (X,u_1) such that $X - u_2u_3(X-B) \subseteq A \subseteq B$.*

Proof. **NECESSARY**: Let A be a semi-closed set of $X, u_1, u_2)$. Then, there exits an open set V in (X, u_1) such that $V \subseteq X - A \subseteq u_2u_3V$. Hence, there exits a closed set B of (X, u_1) such that $V = X - B$ an $X - B \subseteq X - A \subseteq u_2u_3(X - B)$. Therefore, $X - u_2u_3(X - B) \subseteq A \subseteq B$.

SUFFICIENCY: By the assumption, there is a closed set B of (X, u_1) such that $X - u_2u_3(X - B) \subseteq A \subseteq B$. Indeed, there exits an open set V of (X, u_1) such that $B = X - V$ and $X - u_2u_3V \subseteq A \subseteq X - V$. This implies that $V \subseteq X - A \subseteq u_2u_3V$. Therefore, A is semi-closed in (X, u_1, u_2, u_3). □

Theorem 2.2. *Let $\{A_\delta : \delta \in \Delta\}$ be a collection of semi-open sets in a triclosure space (X, u_1, u_2, u_3). Then, $\bigcup_{\delta\in\Delta} A_\delta$ is a semi-open set in (X, u_1, u_2, u_3)*

Proof. Let A_δ be a collection of semi-open sets of (X, u_1, u_2, u_3) for each $\alpha \in \Delta$. Thus, for each $\alpha \in \Delta$, we have an open set V_δ in (X, u_1) such that $V_\delta \subseteq A_\delta \subseteq u_2u_3V_\delta$. Indeed, $\bigcup_{\delta\in\Delta} V_\delta \subseteq \bigcup_{\delta\in\Delta} A_\delta \subseteq \bigcup_{\delta\in\Delta} u_2u_3V_\delta$. Since, $V_\delta \subseteq \bigcup_{\delta\in\Delta} V_\delta$ for each $\delta \in \Delta$, $u_2u_3V_\delta \subseteq u_2u_3 \bigcup_{\delta\in\Delta} V_\delta$ for each $\delta \in \Delta$. Hence, $\bigcup_{\delta\in\Delta} u_2u_3V_\delta \subseteq u_2u_3 \bigcup_{\delta\in\Delta} V_\delta$. In consequence, $\bigcup_{\delta\in\Delta} V_\delta \subseteq \bigcup_{\delta\in\Delta} A_\delta \subseteq u_2u_3 \bigcup_{\delta\in\Delta} V_\delta$. It is well know that V_δ is open in (X, u_1) for each $\delta \in \Delta$, then $u_1 \bigcap_{\delta\in\Delta}(X - V_\delta) \subseteq u_1(X - V_\delta) = X - V_\delta$ for each $\delta \in \Delta$. Thus, $u_1 \bigcap_{\delta\in\Delta}(X - V_\delta) \subseteq \bigcap_{\delta\in\Delta}(X - V_\delta)$. This implies that, $\bigcap_{\delta\in\Delta}(X - V_\delta)$ is closed in (X, u_1), i.e. $\bigcup_{\delta\in\Delta} V_\delta$ is open in (X, u_1). Therefore, $\bigcup_{\delta\in\Delta} A_\delta$ is semi-open in (X, u_1, u_2, u_3). □

The arbitrary intersection of semi-open sets in a triclosure space (X, u_1, u_2, u_3) need not be a semi-open set as can be seen in the following example.

Example 2.2. By the Example 2.1, we can see that $\{a, d\}$ and $\{b, d\}$ are semi-open, but $\{a, d\} \cap \{b, d\} = \{d\}$ is not semi-open.

Theorem 2.3. *Let $\{A_{\delta\in\Delta} : \delta \in \Delta\}$ be a collection of semi-closed set in a triclosure space (X, u_1, u_2, u_3). Then, $\bigcap_{\delta\in\Delta} A_\delta$ is semi-closed in (X, u_1, u_2, u_3).*

Proof. It is clear that the complement of $\bigcap_{\delta\in\Delta} A_\delta$ is $\bigcup_{\delta\in\Delta}(X-A_\delta)$. Since A_δ is semi-closed in (X,u_1,u_2,u_3) for each $\delta\in\Delta$, then $X-A_\delta$ is semi-open for each $\alpha\in\Delta$. But, by the Theorem 2.2, $\bigcup_{\delta\in\Delta}(X-A_\delta)$ is a semi-open set in (X,u_1,u_2,u_3). Therefore, $\bigcap_{\delta\in\Delta} A_\delta$ is semi-closed in (X,u_1,u_2,u_3). □

The arbitrary union of semi-closed sets in a triclosure space (X,u_1,u_2,u_3) need not be a semi-closed set as can be seen in the following example.

Example 2.3. By the Examples 2.1 and 2.2, we can see that $\{a,c\}$ and $\{b,c\}$ are semi-closed, but $\{a,c\}\cup\{b,c\}=\{a,b,c\}$ is not semi-closed.

Theorem 2.4. *Let (X,u_1,u_2,u_3) be a triclosure space and u_2 be idempotent. If A is semi-open in (X,u_1,u_2,u_3) and $A\subseteq B\subseteq u_2u_3A$, then B is semi-open.*

Proof. Let A be a semi-open set of (X,u_1,u_2,u_3). Then, there exits an open set V in (X,u_1) such that $V\subseteq A\subseteq u_2u_3V$, hence $u_2u_3A\subseteq u_2u_2u_3V$. Since u_2 is idempotent, $u_2u_3A\subseteq u_2u_3V$, in consequence $V\subseteq A\subseteq B\subseteq u_2u_3A\subseteq u_2u_3V$. Therefore, B is semi-open. □

Theorem 2.5. *Let $(Y,v_1,v_2.v_3)$ be a triclosure subspace of (X,u_1,u_2,u_3) and $A\subseteq Y$. If A is semi-open in (X,u_1,u_2,u_3), then A is semi-open in (Y,v_1,v_2,v_3).*

Proof. Let A be a semi-open set of (X,u_1,u_2,u_3). Then, there exits an open set V in (X,u_1) such that $V\subseteq A\subseteq u_2u_3V$. This implies that $A\cap Y\subseteq u_2u_3V\cap Y$. But, $A=A\cap Y$, thus $V\subseteq A=A\cap Y\subseteq u_2u_3V\cap Y=u_2u_3V$. Since V is open in (X,u_1), then $v_1(Y-V)=u_1(Y-V)\cap Y\subseteq u_1(X-V)\cap Y=(X-V)\cap Y=Y-V$. Indeed, $Y-V$ is closed in (Y,v_1), i.e. V is open in (X,u_1). Therefore, A is semi-open in (Y,v_1,v_2,v_3). □

In this part, we show some properties on semi-open functions. Throughout this part, $(X,u_1,u_2,u_3),(Y,v_1,v_2,v_3)$ and (Z,w_1,w_2,w_3) are triclosure spaces.

Definition 2.2. Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be a function, then f is called semi-open (respectively, semi-closed) if $f(A)$ is semi-open (respectively, semi-closed) in (Y,v_1,v_2,v_3) for every open (respectively, closed) subset A of (X,u_1,u_2,u_3).

Remark 2.3. It is clear that if a function f is open (respectively, closed), then f is semi-open (respectively, semi-closed). The converse need not be true as can be seen in the following example.

Example 2.4. Let $X = \{a,b,c\} = Y$ and define the closure operator u_1 on X as $u_1\emptyset = \emptyset$, $u_1X = X$, $u_1\{b\} = \{b\}$. Define the closure operator u_2 on X as $u_2\emptyset = \emptyset$, $u_2X = X$, $u_2\{b\} = \{b\}$, $u_2\{b,c\} = \{b,c\}$. Define the closure operator u_3 on X as $u_2\emptyset = \emptyset$, $u_2X = X$, $u_2\{b\} = \{b\}$, $u_2\{b,d\} = \{b,d\}$. We can see that $\{a,c,d\}$ is open in (X,u_1,u_2,u_3). Now, define the closure operator v_1 on Y as $v_1\emptyset = \emptyset$, $v_1X = X$, $v_1\{b,c,d\} = \{b,c,d\}$. Define the closure operator v_2 on Y as $v_2\emptyset = \emptyset$, $v_2X = X$, $v_2\{a,b,c\} = \{a,b,c\}$, $v_2\{a,d\} = \{a,d\}$. Define the closure operator v_3 on Y as $v_3\emptyset = \emptyset$, $v_3X = X$, $v_2\{a,b,d\} = \{a,b,d\}$, $v_3\{b,c\} = \{b,d\}$. Then, we can see that $\{a,c,d\}$ is semi-open, but it is not open in (Y,v_1,v_2,v_3).

Let $f : (X,u_1,u_2,u_3) \to (Y,v_1,v_2,v_3)$ be the identify function. By the above condition, it is easy to see that f is semi-open, but it is not open because $f(\{a,c,d\})$ is not open in (Y,v_1,v_2,v_3) while $\{a,c,d\}$ is open in (X,u_1,u_2,u_3). Furthermore, f is semi-closed, but it is not closed because $f(\{b\})$ is not closed in (Y,v_1,v_2,v_3) while $\{b\}$ is closed in (X,u_1,u_2,u_3).

Theorem 2.6. *Let $f : (X,u_1,u_2,u_3) \to (Y,v_1,v_2,v_3)$ and $g : (Y,v_1,v_2,v_3) \to (Z,w_1,w_2,w_3)$ be two functions. Then, $g \circ f$ is semi-open if f is open and g is semi-open.*

Proof. Let V be an open set of (X,u_1,u_2,u_3) and let f be open, then $f(V)$ is open in (Y,v_1,v_2,v_3). Since g is semi-open, then $g(f(V)) = g \circ f(V)$ is semi-open in (Z,w_1,w_2,w_3). Therefore, $g \circ f$ is semi-open. □

Theorem 2.7. *Let $f : (X,u_1,u_2,u_3) \to (Y,v_1,v_2,v_3)$ and $g : (Y,v_1,v_2,v_3) \to (Z,w_1,w_2,w_3)$ be two functions. If $g \circ f$ is semi-open and f is a continuous surjection, then g is semi-open.*

Proof. Let V be an open set in (Y,v_1,v_2,v_3) and let f be continuous. Then, $f^{-1}(V)$ is open in (X,u_1,u_2,u_3). Since $g \circ f$ is semi-open, $g \circ f(f^{-1}(V))$ is semi-open in (Z,w_1,w_2,w_3). But, it is well known that f is surjection, this implies that $g \circ f(f^{-1}(V)) = g(V)$. Indeed, $g(V)$ is semi-open in (Z,w_1,w_2,w_3). Therefore, g is semi-open. □

3. SEMI-CONTINUOUS AND SEMI-IRRESOLUTE FUNCTIONS IN TRICLOSURE SPACES

In this section, we introduce and study the notions of semi-continuous and semi-irresolute functions obtained by using semi-open sets in triclosure spaces. Throughout this section, $(X,u_1,u_2,u_3),(Y,v_1,v_2,v_3)$ and (Z,w_1,w_2,w_3) are triclosure spaces.

Definition 3.1. Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be a function. Then, f is said to be semi-continuous if $f^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3) for every open set in (Y,v_1,v_2,v_3).

Remark 3.1. It is clear that if a function f is continuous, then f is semi-continuous. But, the converse need not be true as can be seen in the following example.

Example 3.1. Let $X=\{a,b\}=Y$ and define a closure operator u_1 on X as $u_1\emptyset=\emptyset$, $u_1\{a\}=\{a\}$, $u_1\{b\}=u_1X=X$. Define the closure operator u_2 on X as $u_2\emptyset=\emptyset$, $u_2\{a\}=u_2\{b\}=u_2X=X$. Define the closure operator u_3 on X as $u_3\emptyset=\emptyset$, $u_3\{a,b\}=u_3X=u_3\{b\}=X$. Now, define a closure operator v_1 on Y as $v_1\emptyset=\emptyset$, $v_1\{a\}=\{a\}$, $v_1\{b\}=\{b\}$, $v_1X=X$. Define the closure operator v_2 on Y as $v_2\emptyset=\emptyset$, $v_2\{a\}=\{a\}$, $v_2\{b\}=v_2Y=Y$ and define the closure operator v_3 on Y as $v_3\emptyset=\emptyset$, $v_3\{b\}=\{b\}$, $v_3X=X$.

Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be an identity function. We can see that f is semi-continuous but it is not continuous because $f^{-1}(\{b\})$ is not open in (X,u_1,u_2,u_3) while $\{b\}$ is open in (Y,v_1,v_2,v_3).

Proposition 3.1. *A function $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ is semi-continuous if and only if $f^{-1}(B)$ is a semi-closed set of (X,u_1,u_2,u_3) for every closed set of (Y,v_1,v_2,v_3).*

Proof. The proof is followed by the Definition 3.1. □

Theorem 3.1. *Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ and $g:(Y,v_1,v_2,v_3)\to(Z,w_1,w_2,w_3)$ be two functions. If g is continuous and f is semi-continuous, then $g\circ f$ is semi-continuous.*

Proof. Let V be an open set of (Z,w_1,w_2,w_3) and since g is continuous, then $g^{-1}(V)$ is open in (Y,v_1,v_2,v_3). Now, as f is semi-continuous, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, $g\circ f$ is semi-continuous. □

Definition 3.2. A triclosure space (X,u_1,u_2,u_3) is said to be a T_s-space if every semi-open set in (X,u_1,u_2,u_3) is open in (X,u_1,u_2,u_3).

Theorem 3.2. *Let* $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ *and* $g:(Y,v_1,v_2,v_3)\to(Z,w_1,w_2,w_3)$ *be two functions and* (Y,v_1,v_2,v_3) *be a* T_s*-space. If* f *and* g *are semi-continuous, then* $g\circ f$ *is semi-continuous.*

Proof. Let V be an open set of (Z,w_1,w_2,w_3). Since g is semi-continuous, $g^{-1}(V)$ is semi-open in (Y,v_1,v_2,v_3). But, we know that (Y,v_1,v_2,v_3) is a T_s-space, thus $g^{-1}(V)$ is open in (Y,v_1,v_2,v_3). As f is semi-continuous, then $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, $g\circ f$ is semi-continuous. □

Theorem 3.3. *Let* $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ *and* $g:(Y,v_1,v_2,v_3)\to(Z,w_1,w_2,w_3)$ *be two functions. Then, the following statements hold:*

(1) If f *is a semi-open surjection and* $g\circ f$ *is continuous, then* g *is semi-continuous.*
(2) If g *is a semi-continuous injection and* $g\circ f$ *is open, then* f *is semi-open.*
(3) If g *is an open injection and* $g\circ f$ *is semi-continuous, then* f *is semi-continuous.*

Proof. (1) Let V be an open set of (Z,w_1,w_2,w_3) and $g\circ f$ be continuous. Then, $(g\circ f)^{-1}(V)$ is open in (X,u_1,u_2,u_3). Since f is a semi-open function, then $f(g\circ f)^{-1}(V))=f(f^{-1}(g^{-1}(V)))$ is semi-open in (Y,v_1,v_2,v_3). But, we know that f is a surjection, indeed $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$. Therefore, g is semi-continuous.

(2) Let V be an open set of (X,u_1,u_2,u_3) and $g\circ f$ be open. Then, $g\circ f(V)$ is open in (Z,w_1,w_2,w_3). Since g is semi-continuous, then $g^{-1}(g\circ f)(V))$ is semi-open in (Y,v_1,v_2,v_3). But, we know that g is an injection, indeed $g^{-1}(g\circ f(V))=f(V)$. Therefore, f is semi-open.

(3) Let V be an open set of (Y,v_1,v_2,v_3) and g be open. Then, $g(V)$ is open in (Z,w_1,w_2,w_3). Since $g\circ f$ is semi-continuous, then $(g\circ f)^{-1}(g(V))$ is semi-open in (X,u_1,u_2,u_3). But, we know that g is an injection, this implies that $(g\circ f)^{-1}(g(V))=f^{-1}(g^{-1}(g(V)))=f^{-1}(V)$. Therefore, f is semi-continuous.

□

Definition 3.3. Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be a function. Then, f is said to be semi-irresolute if $f^{-1}(V)$ is a semi-open set of (X,u_1,u_2,u_3) for every semi-open set V of (Y,v_1,v_2,v_3).

Theorem 3.4. *If a function $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ is semi-irresolute, then f is semi-continuous.*

Proof. Let V an open set of (Y,v_1,v_2,v_3), then it is well known that every open set is semi-open, indeed V is semi-open in (Y,v_1,v_2,v_3), since f is semi-irresolute, we have that $f^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, f is semi-irresolute. □

The following example shows that the converse of the above Theorem need not be true.

Example 3.2. Let $X=\{a,b\}=Y$ and define the closure operator u_1 on X as $u_1\emptyset=\emptyset$, $u_1\{a\}=u_1\{b\}=u_1X=X$. Define the closure operator u_2 on X as $u_2\emptyset=\emptyset$, $u_2\{a\}=u_2\{b\}=u_2X=X$. Define the closure operator u_3 on X as $u_3\emptyset=\emptyset$, $u_3X=X$ and $u_3A=A$, where $A\subset X$. Now, define the closure operator v_1 on Y as $v_1\emptyset=\emptyset$, $v_1\{a\}=\{a\}$, $v_1\{b\}=v_1Y=Y$. Define the closure operator v_2 on Y as $v_2\emptyset=\emptyset$, $v_2\{a\}=v_2\{b\}=v_2Y=Y$ and define the closure operator v_3 on Y as $v_3\emptyset=\emptyset$, $v_3X=X$ and $v_3B=B$, where $B\subset X$.

Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be an identity function. Then, we can see that there are only two open sets in (Y,v_1,v_2,v_3) which are $\emptyset$ and Y and their inverse images are semi-open in (X,u_1,u_2,u_3). Indeed, f is semi-continuous. But, f is not semi-irresolute because $f^{-1}(\{b\})$ is not semi-open in (X,u_1,u_2,u_3) while $\{b\}$ is semi-open in (Y,v_1,v_2,v_3).

Theorem 3.5. *Let $f:(X,u_1,u_2,u_3)\to(Y,v_1,v_2,v_3)$ be an open, semi-irresolute and surjective function. Then, (Y,v_1,v_2,v_3) is a T_s-space if (X,u_1,u_2,u_3) is a T_s-space.*

Proof. Let (X,u_1,u_2,u_3) be a T_s-space and let V be a semi-open set of (Y,v_1,v_2,v_3). Since f is semi-irresolute, $f^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). We know that (X,u_1,u_2,u_3) is a T_s-space, then $f^{-1}(V)$ is open in (X,u_1,u_2,u_3). Now, since f is open, then $f(f^{-1}(V))$ is open in (Y,v_1,v_2,v_3). But, we know that f is a surjection, hence $f(f^{-1}(V))=V$. In consequence, V is open in (Y,v_1,v_2,v_3). Therefore, (Y,v_1,v_2,v_3) is a T_s-space. □

Theorem 3.6. *Let* $f:(X,u_1,u_2,u_3)\rightarrow(Y,v_1,v_2,v_3)$ *and* $g:(Y,v_1,v_2,v_3)\rightarrow(Z,w_1,w_2,w_3)$ *be two functions. Then, the following statements hold:*

(1) $g\circ f$ *is semi-continuous, if* f *is semi-irresolute and* g *is semi-continuous.*

(2) $g\circ f$ *is semi-irresolute, if* f *is semi-irresolute and* g *is semi-irresolute.*

(3) $g\circ f$ *is semi-continuous, if* f *is semi-continuous and* g *is continuous.*

Proof. (1) Let V be an open set of (Z,w_1,w_2,w_3) and since g is semi-continuous, then $g^{-1}(V)$ is semi-open in (Y,v_1,v_2,v_3). Now, as f is semi-irresolute, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, $g\circ f$ is semi-continuous.

(2) Let V be a semi-open set of (Z,w_1,w_2,w_3) and since g is semi-irresolute, then $g^{-1}(V)$ is semi-open in (Y,v_1,v_2,v_3). Now, as f is semi-irresolute, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, $g\circ f$ is semi-irresolute.

(3) Let V be an open set of (Z,w_1,w_2,w_3) and since g is continuous, then $g^{-1}(V)$ is open in (Y,v_1,v_2,v_3). Now it is well known that every open set is semi-open, indeed $g^{-1}(V)$ is semi-open in (Y,v_1,v_2,v_3), as f is semi-irresolute, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is semi-open in (X,u_1,u_2,u_3). Therefore, $g\circ f$ is semi-continuous.

□

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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