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Research Article

Non-Automatic Weighing Instruments Calibration: Parametric Methods for Outliers Assessments

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Abstract

In the experimental calibration processes, several error of measure can be obtained. Usually, these values are called as outliers. The outliers impact in the instruments' metrological reliability and, consequently, in the industrial processes' quality control. In specific terms, for mass metrology, the outliers are extremely critic, due to sensibility of technology used for this industry, i.e.: non-automatic weighing instruments (scales). The EURAMET/cg-18/v.02 is the international standard that defined procedure and tests for to assessment the metrological reliability of scales, however, a statistical treatment for outliers is not defined. In order to propose robust alternatives for outliers analyses, this work to evaluate parametric techniques for the elimination of outliers in the calibration of a Bernalo non-automatic weighing instruments (Capacity: 30.000 kg; Resolution: 0.001 kg). Three methods were applied in the analysis of experimental data: Dixon, Grubbs and Chauvenet. The results confirmed a reduction in the expanded uncertainty associated to mass measurement (k=2) up to: 17.6% (Dixon); 19.9% (Grubbs) and 35.2% (Chauvenet). According to the results and considering the contribution of work to applied mass metrology, the authors propose the inclusion of analyzes and evaluation of outliers in the EURAMET/cg-18/v.02. Thus, it is expected an increase in metrological reliability in mass measurement processes at the scientific and industrial sector.

Keywords: Applied Metrology, Metrological Reliability, Outliers, Non-Automatic Weighing Instruments, Calibration.

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1. Introduction

The measurement of mass is an important physical quantity for laboratories and industrial process that must be controlled from non-automatic weighing instruments (scale). The International Organization of Legal Metrology (OIML) R-76-1 [1] define a procedure of calibration for this technology. This document states that five tests are necessary to calibrate a scale, as: repeatability, eccentric loading, weight, auxiliary verification device and substitution of standard weights at verification. Even though, this procedure is common and applied for different metrology laboratories in many countries, it has some technical and economics disadvantages for industries processes, for example: an industrial process is controlled by a digital scale (Cap.: 300 kg). The maximum weight in the process is 200 kg. The scale must be calibrated up to 300 kg despite the fact that the scale is not used in this range. The economic impact in this case is worse for smaller companies. Thus, since 2009 the European Association of National Metrology Institutes (EURAMET) has published an international recommendation to calibrate scales, as: EURAMET/cg-18/v.02 [2] that allows some flexibilities for industrial processes, *e.g.*: a scale could be calibrate up to the measurement range that it is used in the industrial process.

Nevertheless, both recommendations (OIML R-76-1 and EURAMET/cg-18/v.02) do not take into consideration an important and undesirable factor that could appear during the calibration process called outliers. These values tend to increase a measurement uncertainty, systematic error and allow that a determined instrument will not be according to the international standard [3, 4].

Therefore, the aim of this paper is to analyses metrologically three parametric approaches to evaluate outliers [5] in the scales calibration: Dixon, Chauvenet and Grubbs. Moreover, the results are expected to contribute for the advancement of knowledge of applied mass metrology.

2. Theoretical Fundamentals

This paper proposes to study the performance of three different parametric methods for to eliminate outliers. In order to get this objective, the principal assumption is a Gaussian behavior of the experimental data. Thus, it was possible to apply these methods.

2.1. Dixon test
Dixon test is a parametric method to eliminate outliers. This method consider a test statistic Q_{cal} and compare it with a tabulated critical value (Q_{crit}). If $Q_{cal} \geq Q_{crit}$ the suspect value

must be eliminated. For $8 \leq n \leq 12$, Q_{cal} can be calculated as in the Eq. 1:

$$Q = \frac{x_2 - x_1}{x_{n-1} - x_1} \qquad (1)$$

2.2. Chauvenet test

This method establishes that a x_i value measured must be eliminated when r parameter is greater that R_c which it is tabulated for Chauvenet test. The r parameter can be calculated applying Eq. 2:

$$r = \frac{|x_i - \bar{x}|}{s(x)} \qquad (2)$$

2.2. Grubbs test

For an experiment which has a data series g_i for $i = 1, 2,…, n$, configured ascendingly, the equation (3) is applied to detect outliers:

$$G_c = \frac{(g_i - \bar{g})}{s} \qquad (3)$$

The G_c parameter is compared with a critical value (G_{crit}) which it is tabulated for Grubbs test, considering a specifically significance level. Outliers are detected when $G_c > G_{crit}$.

3. Experimental Methodology

This paper evaluate the outliers in a calibration process for a digital scale that is illustrated in Fig. 1.

Fig. 1. Bernalo Non-Automatic Weighing Instrument (Capacity: 30.000 kg; Resolution: 0.001 kg)

At the beginning of the calibration process, it was measured 11 experimental points in the measuring interval (*i.e.*: total scale capacity). In addition, it was performed 10 repetitions for each experimental point. Besides, atmospheric pressure and environment temperature were measured, respectively, with a barometer (U=0.058 mbar/abs; k=2) and thermometer (U=0.25 °C; k=2) calibrated in an accredited metrology laboratory [6, 7]. Thus, it was calculated the local air density and the buoyancy factor. Then, it was estimated the sample mean and standard deviation. Then, parametric methods to eliminated outliers were applied on the experimental data. After that, the experimental data were fitted for a polynomial curve and the expanded uncertainty was calculated for a confidential level equal to 95.0%. Finally, authors compared the reduction the expanded uncertainty for the three different methods applied (Dixon, Chauvenet and Grubbs) against the original data.

4. Results and Discussion

Applying the experimental methodology described in section 3, it was possible to realize a statistical analyze. The follows sections show the principal results.

4.1. Outliers and measurement uncertainty analyses

Tab. 1 shows the experimental data for each experimental calibration point. Even though, the EURAMET/cg-18/v.02 recommend five experimental point, in order to reduce the measurement uncertainty, in this work was taken ten experimental point. Furthermore, table 1 illustrates some values in yellow highlight that represent the different outliers. These values increase the measurement uncertainty and, probably, the instrument will not be according to the measuring process. In fact, Chauvenet is the most powerful test because the statistical analyses detected and eliminated ten outliers (all the value in yellow highlight). The same way, Grubbs test detected and eliminated seven outliers (*i.e.*: 2.002; 2.998; 5.998; 9.002; 20.004 and 29.998 kg). In contrast, Dixon test eliminated only five outliers and, for this reason, its performance was the weakest. The outliers eliminated through Dixon test are: 2.998; 9.002; 20.004; 22.002 and 29.998 kg.

Table 1. Experimental data of the calibration process

Experimental Calibration Point	Standard Mass	Experimental Data (Measurement)									
		#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
	kg	kg	kg	kg	kg	kg	kg	kg	kg	kg	kg
1	0	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.002	0.003
2	1	1.003	1.005	1.003	1.003	1.003	1.003	1.003	1.005	1.005	1.003
3	2	2.002	2.004	2.004	2.004	2.004	2.004	2.004	2.004	2.004	2.003
4	3	2.998	2.996	2.996	2.996	2.996	2.996	2.996	2.996	2.996	2.996
5	4	4.001	4.001	4.001	4.001	4.001	4.001	4.003	4.003	4.003	4.001
6	5	5.000	5.000	5.000	5.000	5.000	4.999	4.999	4.999	4.999	4.999
7	6	5.998	5.997	5.995	5.995	5.995	5.995	5.995	5.995	5.995	5.995
8	7	7.007	7.006	7.007	7.007	7.007	7.007	7.007	7.006	7.006	7.006
9	8	7.992	7.995	7.992	7.992	7.992	7.992	7.992	7.995	7.995	7.995
10	9	9.001	9.001	9.001	9.001	9.001	9.001	9.001	9.001	9.001	9.002
11	10	10.008	10.008	10.008	10.008	10.008	10.008	10.008	10.008	10.009	10.009
12	20	20.002	20.002	20.002	20.002	20.002	20.002	20.002	20.002	20.002	20.004
13	22	22.001	22.001	22.001	22.001	22.001	22.001	22.001	22.001	22.001	22.002
14	25	24.996	24.998	24.996	24.996	24.996	24.996	24.996	24.996	24.996	24.998
15	30	29.997	29.997	29.997	29.997	29.997	29.997	29.997	29.997	29.997	29.998

Using the result of the above table, it was possible applied Eq. 4 for to calculate the sample mean ($\bar{x}$) and Eq. 5 for the standard deviation (**s**):

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \tag{4}$$

$$s = \left[\frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{\frac{1}{2}} \tag{5}$$

In order to evaluate the repeatability, the uncertainty type A (**u_A**) was calculated applied Eq. 6:

$$u_A = \frac{s}{\sqrt{n}} \tag{6}$$

The literature [6, 7] shows that when outliers are eliminated of the experimental data, the **u_A** is reduced up to 31.2%. The results in this work confirm that the expanded uncertainty (**U_E**) can also be reduced applying parametric methods to eliminate outliers.

In relation to environment temperature (**T_{air}**), it was measured with a calibrated thermometer and its uncertainty (**U_{Tair} = 0.25 °C**) is reported at the certificate of calibration. The atmospheric pressure (**P_{atm}**) was measured with a barometer calibrated and the uncertainty (**U_{Patm} = 0.058 mbar/abs**) is reported at the certificate of calibration. Finally, the local air density (**ρ_{ar}**) was calculated applying the ideal gas law as in Eq. 7:

$$\rho_{air} = \frac{P_{atm}}{R_{air} \cdot T_{air}} \tag{7}$$

The expansion series of Taylor described in [8] was applied in the Eq. 7 and the mathematical development is described below:

$$U_{\rho_{air}}^2 = \left(\frac{\partial \rho_{air}}{\partial P_{atm}} \cdot U_{P_{atm}} \right)^2 + \left(\frac{\partial \rho_{air}}{\partial R_{air}} \cdot U_{R_{air}} \right)^2 + \left(\frac{\partial \rho_{air}}{\partial T_{air}} \cdot U_{T_{air}} \right)^2 \tag{8}$$

In the expression above, the sensibility coefficients are calculated applying equations 9, 10 and 11:

$$\frac{\partial \rho_{air}}{\partial P_{atm}} = \frac{1}{R_{air} \cdot T_{air}} \tag{9}$$

$$\frac{\partial \rho_{air}}{\partial R_{air}} = 0 \quad \therefore$$
$$R_{air} : Constant \ of \ the \ air \ (ideal \ gas) \tag{10}$$

$$\frac{\partial \rho_{air}}{\partial T_{air}} = - \frac{P_{atm}}{R_{air} \cdot T_{air}^2} \tag{11}$$

Replacing equations 9, 10 and 11 in Eq. 8 it was possible obtained Eq. 12:

$$U_{\rho_{air}}^2 = \left(\frac{1}{R_{air} \cdot T_{air}} \cdot U_{P_{atm}} \right)^2 + \left(- \frac{P_{atm}}{R_{air} \cdot T_{air}^2} \cdot U_{T_{air}} \right)^2 \tag{12}$$

Replacing Eq. 7 in Eq. 12 obtained the expression for the expanded uncertainty associated to density of the air:

$$\left(\frac{U_{\rho_{air}}}{\rho_{air}} \right)^2 = \left(\frac{U_{P_{atm}}}{P_{atm}} \right)^2 + \left(\frac{U_{T_{air}}}{T_{air}} \right)^2 \tag{13}$$

Thus, Tab. 2 summarizes the results obtained in term of the sample mean, standard deviation, environment temperature, atmospheric pressure, air density and their associated uncertainties.

Table 2. Calculate of parameters involved in the measurement of mass

Sample Mean	Standard Deviation	Environment Temperature	Uncertainty	Atmospheric Pressure	Uncertainty	Density of air	Uncertainty
$\bar{x}$	s	T_{air}	U_{Tair}	P_{atm}	U_{patm}	ρ_{ar}	$U_{\rho ar}$
kg	kg	°C	°C	mbar/abs	mbar/abs	kg/m³	kg/m³
0.001	0.00070	27.9	0.25	1019.8	0.058	1.180	0.0010
1.004	0.00097	27.9	0.25	1019.8	0.058	1.180	0.0010
2.004	0.00067	27.9	0.25	1019.8	0.058	1.180	0.0010
2.996	0.00063	27.9	0.25	1019.8	0.058	1.180	0.0010
4.002	0.00097	27.9	0.25	1019.8	0.058	1.180	0.0010
5.000	0.00053	27.9	0.25	1019.8	0.058	1.180	0.0010
5.996	0.00108	27.9	0.25	1019.8	0.058	1.180	0.0010
7.007	0.00052	27.9	0.25	1019.8	0.058	1.180	0.0010
7.993	0.00155	27.9	0.25	1019.8	0.058	1.180	0.0010
9.001	0.00032	27.9	0.25	1019.8	0.058	1.180	0.0010
10.008	0.00042	27.9	0.25	1019.8	0.058	1.180	0.0010
20.002	0.00063	27.5	0.25	1019.8	0.058	1.182	0.0010
22.001	0.00032	27.5	0.25	1019.8	0.058	1.182	0.0010
24.996	0.00084	27.5	0.25	1019.8	0.058	1.182	0.0010
29.997	0.00032	27.5	0.25	1019.8	0.058	1.182	0.0010

4.2. Metrological comparison: Dixon, Chauvenet and Grubbs

After remove the outliers of the original data (Tab. 1) for the Dixon, Chauvenet and Grubbs methods, it was possible to applying a statistics treatment for estimated the expanded uncertainty for four different situations: (i) *original data*; (ii) *data after applying Dixon method*; (iii) *data after applying Chauvenet method* and (iv) *data after applying Grubbs method*. This work considered four source of uncertainty to calculate the expanded uncertainty associated to measurement of mass: standard mass, resolution of instruments, uncertainty of polynomial fit and repeatability. The reference [5] shows more details about the uncertainty analyses.

In order to show the principal results of the uncertainty analyses, the Tab. 3 summarize performing a comparison among Dixon, Chauvenet and Grubbs test. Therefore, the Fig. 2 illustrate an important reduction for each method when compared to the original data.

Table 3. Measurement uncertainty

| Indicated mass	Expanded Uncertainty (U$_E$)				Measurement Uncertainty Reduction (MUR)		
	Original data	Dixon	Chauvenet	Grubbs	Dixon	Chauvenet	Grubbs
kg	kg	kg	kg	kg	%	%	%
0.001	0.0024	0.0024	0.0021	0.0024	0.0%	11.7%	0.0%
1.004	0.0027	0.0027	0.0027	0.0027	0.0%	0.0%	0.0%
2.004	0.0023	0.0023	0.0019	0.0023	0.0%	19.4%	0.0%
2.996	0.0023	0.0019	0.0019	0.0019	17.6%	17.6%	17.6%
4.002	0.0027	0.0027	0.0027	0.0027	0.0%	0.0%	0.0%
5.000	0.0022	0.0022	0.0022	0.0022	0.0%	0.0%	0.0%
5.996	0.0029	0.0029	0.0019	0.0023	0.0%	35.2%	19.9%
7.007	0.0022	0.0022	0.0022	0.0022	0.0%	0.0%	0.0%
7.993	0.0037	0.0037	0.0037	0.0037	0.0%	0.0%	0.0%
9.001	0.0020	0.0019	0.0019	0.0019	5.4%	5.4%	5.4%
10.008	0.0021	0.0021	0.0021	0.0021	0.0%	0.0%	0.0%
20.002	0.0023	0.0019	0.0019	0.0019	17.6%	17.6%	17.6%
22.001	0.0020	0.0019	0.0019	0.0019	5.4%	5.4%	5.4%
24.996	0.0026	0.0026	0.0023	0.0026	0.0%	9.0%	0.0%
29.997	0.0020	0.0019	0.0019	0.0019	5.4%	5.4%	5.4%
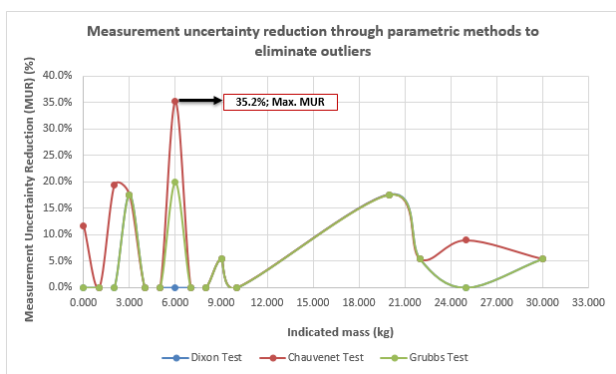
Fig. 2. Measurement uncertainty

Tab. 3 and Fig. 2 confirm that the maximum measurement reduction was offered by Chauvenet test (35.2%). In addition, Dixon test offered a maximum uncertainty reduction in 17.6% and Grubbs for 19.9%. In fact, all the parametric method for exclusion outliers applied in this work offered an important expanded uncertainty reduction when compared with the original data uncertainty show in Tab. 3.

5. Conclusions

This paper compared three different parametric method to eliminate outliers. These values increase the measurement uncertainty and produce systematic errors. The consolidated results confirmed that the assessed methods (Dixon, Chauvenet and Grubbs) constitute a significant strategy to reduce the expanded measurement uncertainty up to 35.2%, into a confidence level of 95.0%. Finally, this paper propose to include a revision and statistical analyses of outliers in the EURAMET/cg-18/v.02 and, consequently, in the *Guía* SIM (2009).

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